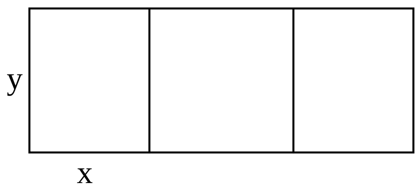
Calculus 1 Name______________________
Additional Problems with Optimization Date______________________

1) A rectangular pen with three equal sections made by parallel partitions is to be constructed using 500 feet of fencing. What dimensions will maximize the total area of the pen? What are the dimensions of each pen?



Maximize the Area

A=xy or A=3xy

You need to get rid of either the x or y, so you use → 6x+4y=500

$$y = \frac{500}{4} - \frac{6x}{4}$$

$$y = 125 - \frac{3x}{2}$$

$$A = x\left(125 - \frac{3x}{2}\right)$$

$$A = 125x - \frac{3x^2}{2}$$

You can now take the derv. to find the maximum but finding the zeros of the derv.

$$A' = 125 - 3x$$

$$0 = 125 - 3x$$

$$x = 41.667$$

Now you need to find y.

$$y = 125 - 3 * \frac{41.667}{2}$$

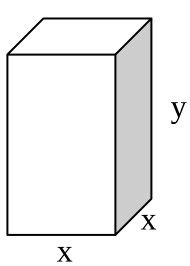
$$y = 62.5$$

The dimensions of one pen are 41.667ft by 62.5ft.

The dimensions of the total pens are 125.001ft by 62.5ft

(the x value needed to be multiplied by 3)

2) An open rectangular box with a square base is to be made from 48 square feet of material. What dimensions will result in a box with the largest possible volume?



Since this is a square base the width & length can be x & the height is y.

$$V = x^2 y$$

Need to get rid of y or x.

$$V = x^2\left(-\frac{x^2}{4x} + \frac{48}{4x}\right)$$

$$V = -\frac{x^3}{4} + 12x$$

Take the derv.

48 square feet is surface area

base + 4*sides = Surface Area

$$x^2 + 4xy = 48$$

$$4xy = -x^2 + 48$$

$$y = -\frac{x^2}{4x} + \frac{48}{4x}$$

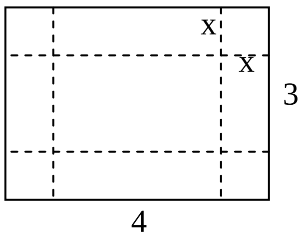
$$V' = -\frac{3}{4}x^2 + 12$$

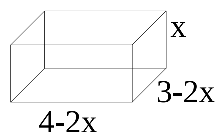
x = 4

$$y = -\frac{4^2}{4*4} + \frac{48}{4*4} = 2$$

Find y & write your sentence. "The base is 4ft by 4ft & the height is 2ft."

3) A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What dimensions of the box are needed to produce a box with the largest volume?





Make the small piece after the fold "x".

The remaining sides can then be found and the height is x.

$V = (4 - 2x)(3 - 2x)(x)$

$V = (12 - 6x - 8x + 4x^2)(x)$

$V = 12x - 14x^2 + 4x^3$

Take the derv. to find the x value of the maximum

$V' = 12 - 28x + 12x^2$

$0 = 12 - 28x + 12x^2$

$x = .566, 1.768$

Can't use 1.768 because it is a min. & the one side is 3 and half of 3 is 1.5, so 1.768 is too big.

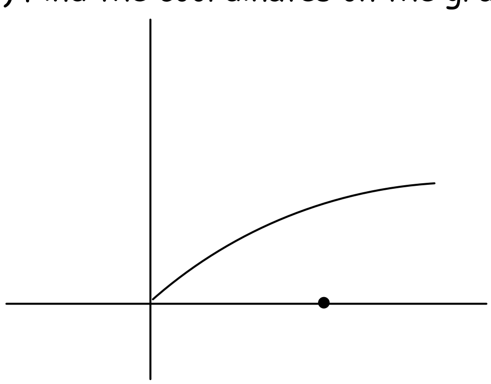
Plug in x to find the other two sides.

4-2(,566)=2.868ft Length

3-2(.566)=1.868ft Width

.566ft Height

4) Find the coordinates on the graph of $y = \sqrt{x}$ that is nearest to the point (4, 0).



You need to minimize the distance between the line and the point (4,0)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x - 4)^2 + (y - 0)^2}$

$d = \sqrt{(x - 4)^2 + y^2}$

You need to use the original equation to get rid of y

$d = \sqrt{(x - 4)^2 + (\sqrt{x})^2}$

$d = \sqrt{(x - 4)^2 + x}$

Now you can take the derivative.

$d' = \frac{1}{2}((x - 4)^2 + x)^{-\frac{1}{2}}(2(x - 4) + 1)$

$0 = \frac{1}{2}((x - 4)^2 + x)^{-\frac{1}{2}}(2(x - 4) + 1)$

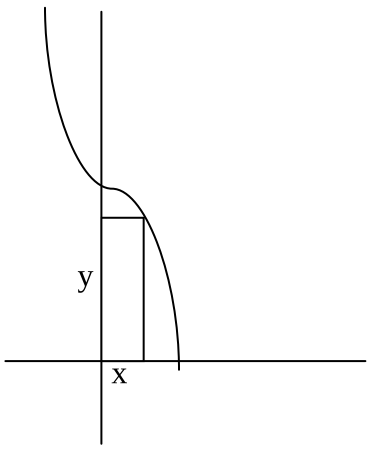
$x = 3.5$

To find the y value use the original equation.

$y = \sqrt{3.5} = 1.871$

The point closest to the graph is (3.5, 1.871).

5) A rectangle is to be inscribed in the closed region bounded by the x-axis, y-axis, and graph of $y = 8 - x^3$. What are the dimensions of the rectangle if the area is to be maximized?



Maximize the Area

A=xy

You need to get rid of x or y. Use original equation!

$A = x(8 - x^3)$

$A = 8x - x^4$

Take the derv.

$A' = 8 - 4x^3$

$0 = 8 - 4x^3$

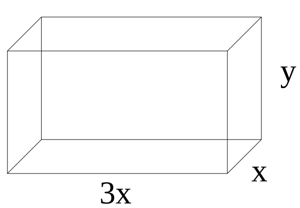
$x = 1.26$

Plug this in to find the y value.

$y = 8 - (1.26)^3 = 6$

The rectangle has dimensions 1.26 by 6.

6) A box is to be constructed where the base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.



Minimize the Cost which is based of square feet (Area)

Cost for <u>Base&Top</u> + Cost for <u>Front&Back</u> + Cost for <u>Right&Left</u>

$10(2)(3x^2) + 6(2)(3xy) + 6(2)(xy) = \text{Total Cost}$

Simplify

$60x^2 + 36xy + 12xy = C$

$60x^2 + 48xy = C$

Need to get rid of x or y. So use volume equation. $50 = 3x^2y$

$\frac{50}{3x^2} = y$

$60x^2 + 48x\left(\frac{50}{3x^2}\right) = C$

$60x^2 + 800x^{-1} = C$

Take the derv. and find the zeros to find the minimum.

$120x - 800x^{-2} = C'$

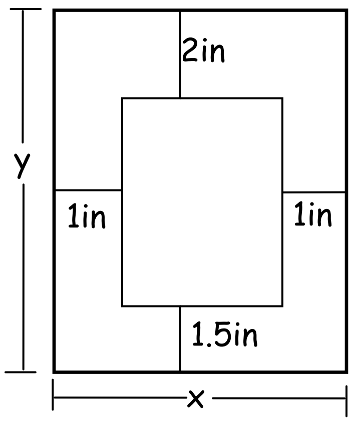
$x = 1.882$

Use x = 1.882 to find the other two sides.

One side is 1.822ft, the other side is 3x which is 5.646ft, & the height is $\frac{50}{3x^2}$ which is 4.706ft.

1.822 ft by 5.646 ft by 4.706 ft

7) A printer need to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions of the poster that will give the largest printed area? What are the dimensions of the printed area?



Maximize the printed area.

$$A_{\text{printed area}} = (x-2)(y-3.5)$$

Use total area equation to get x or y alone

$$A_{\text{entire poster}} = xy$$
$$200 = xy$$
$$\frac{200}{x} = y$$

$$A_{\text{printed area}} = (x-2)(y-3.5)$$
$$A = (x-2)\left(\frac{200}{x} - 3.5\right)$$
$$A = 200 - 3.5x - 400x^{-1} + 7$$

Take the derv. and find the zeros to find the maximum

$$A' = -3.5 + 400x^{-2}$$
$$0 = -3.5 + 400x^{-2}$$

x = 10.69

Use x = 10.69 to find the other side.

$$y = \frac{200}{10.69} = 18.709$$

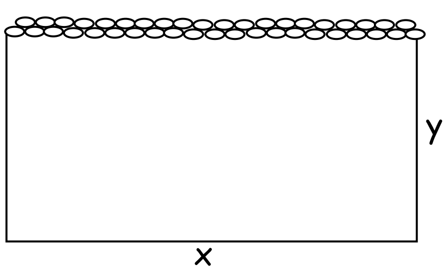
Entire Poster is 10.69in by 18.709in

One printed side is (x-2) which is 8.69in,

the other printed side is (y-3.5) which is 15.29in

Printed Poster is 8.69in by 15.29 in

8) A rectangular animal pen is to be constructed so that one wall is against an existing stone wall and the other three sides are to be fence. If 500 feet of fence is to be used, determine the dimensions and area of the pen with maximal area.



Maximize the Area

$$A = xy$$

Use the fencing equation to solve for x or y.

$$F = 2y + x$$
$$500 = 2y + x$$
$$500 - 2y = x$$

$$A = (500 - 2y)y$$
$$A = 500y - 2y^2$$

Take the derv and set equal to zero to find the maximum.

$$A' = 500 - 4y$$
$$0 = 500 - 4y$$
$$y = 125$$

Use fencing equation to solve for x.

$$x = 500 - 2(125) = 250$$

The dimensions are 250ft by 125ft