18 SEPTEMBER 2015 THEORY

1 o

y’ = y + t, 0 ≤ t ≤ 0.1, y (t) = −1 + e^t − t

```
DSolve[{y'[t] == y[t] + t, y[0] == 0}, y[t], t]
{{y[t] → -1 + e^t - t}}
Clear[t]
g[t_] := -1 + e^t - t;
Simplify[D[g[t], t] - g[t] - t]
y = g[0];
Print["Initial value y0 = ", y]
0
Initial value y0 = 0
```

RK4

```
f[t_, y_] := y + t;
a = 0; b = 0.1; n = 1; l = 0.1; t = 0;
x2 = N[g[t]]; x3 = Abs[y - x2]; y = g[0];
Print["Initial value y0 = ", x2];
Do[Print["STEP  : ", i]; k1 = f[t, y]; Print["k1 = ", N[
      k2 = f[t + l/2, y + l k1/2]; Print["k2 = ", N[k2, 7]];
      k3 = f[t + l/2, y + l k2/2]; Print["k3 = ", N[k3, 7]];
      k4 = f[t + l, y + l k3]; Print["k4 = ", N[k4, 7]];
      x = y + 1/6 l (k1 + 2 k2 + 2 k3 + k4);
      t = t + l;
      t1 = N[t];
      x1 = N[x];
      y = x; x2 = N[g[t]]; x3 = Abs[x1 - x2];
      Print["time : ", t1, ", numerical : ", x1, ", th
```

Initial value y_0 = 0.

STEP : 1

k_1 = 0

k_2 = 0.05

k_3 = 0.0525

k_4 = 0.10525

time : 0.1, numerical : 0.005170833
 , theoretical : 0.005170918, error : 8.474231×10^{-8}

2 o

ii)

COMPOSITE SIMPSON

```
Clear[x]
f[x_] := 1 / Sqrt[1 + x^4]
n = 6; a = 0; b = 0.6; h = 0.1;
Print[a, "   ,   ", f[a]]
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0; Sb1 = 0; Sb2 = 0;
Do[a += h; If[EvenQ[i], Sb2 += f[a], Sb1 += f[a]]; Print[a
Print[b, "   ,   ", f[b]]
```

$$Sb = \frac{1}{3} h (Sb0 + 4 Sb1 + 2 Sb2);$$

```
Print["Composite Simpson I(f)=", N[Sb]]
```

```
0   ,   1
0.1   ,   0.99995
0.2   ,   0.999201
0.3   ,   0.9959744
0.4   ,   0.9874406
0.5   ,   0.9701425
0.6   ,   0.9408874
Composite Simpson I(f)=0.5926146
```

3 o

i)

```
f[x_] := Log[1 + x^4]
x0 = 1; x1 = 1.5; x2 = 2;
data = {{x0, f[x0]}, {x1, f[x1]}, {x2, f[x2]}};
P[x_] := InterpolatingPolynomial[data, x]
Expand[N[P[x]]]
```

$-1.758455 + 2.60737\,x - 0.155768\,x^2$

ii)

LEAST SQUARES METHOD

```
data = {{1, 2}, {1.5, 2.5}, {2, -1.5}, {3, 1}};
Fit[data, {1, x, x^2}, x]
```

$9.318182 - 8.572727\,x + 1.909091\,x^2$

18 SEPTEMBER 2015 LABORATORY

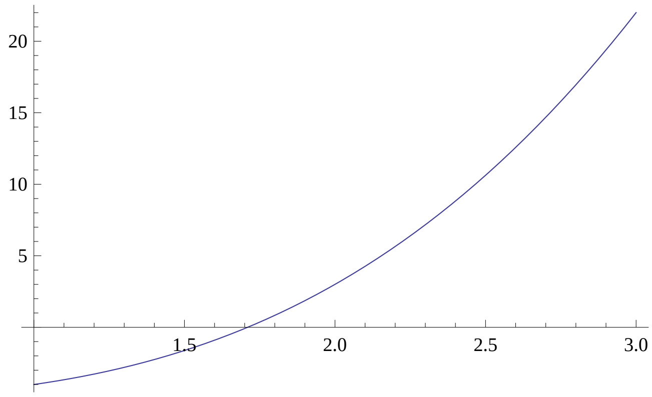
1 o

In[1]:=
```
Clear[x]
f[x_] := x^3 - 5
D[f[x], x]
Print["f(x)=", f[x], ",   f'(x)=", D[f[x], x]]
Plot[f[x], {x, 1, 3}]
Clear[x]
g[x_] := Together[x - f[x] / (3 x^2)]
Print["x-f(x)/f'(x)  = ", g[x]]
x = 1.5;
Print["Initial value : ", x]
Do[y = g[x];
 Print[i, "   ,   ", N[y, 10]]; x = y, {i, 1, 3}]
Print["Theoretical value : ", N[5^(1/3)]]
```

Out[3]= $3 x^2$

$f(x) = -5 + x^3, \quad f'(x) = 3 x^2$

Out[5]=



$x - f(x)/f'(x) \ = \ \frac{5 + 2 x^3}{3 x^2}$

```
Initial value : 1.5
1   ,   1.740741
2   ,   1.710516
3   ,   1.709976
Theoretical value : 1.709976
```

2 o

Theoretical Value

```
Clear[x]
f[x_] := Sqrt[1 + x^2]
th = Integrate[f[x], {x, 0, 1}];
Print["Theoretical value = ", th,
 ",  Approximation : ", N[th]]; Print[" "];
```

Theoretical value = $\frac{1}{2}\left(\sqrt{2} + \text{ArcSinh}[1]\right)$

, Approximation : 1.147794

Numerical Value

COMPOSITE TRAPEZOIDAL RULE

```
Clear[x];
f[x_] := Sqrt[1 + x^2];
a = 0; b = 1; h = 0.1; n = (b - a)/h;
TV = NIntegrate[f[x], {x, a, b}];
Print["Theoretical value =", TV]; Print[" "];
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0; Sb1 = 0; Sb2 = 0;
Do[a += h; Str1 += f[a], {i, 1, n - 1}]
Str = 1/2 h (Str0 + 2 Str1);
Print["Composite Trapezoidal I(f) =", N[Str], "    Absc
```

```
Theoretical value =1.147794

Composite Trapezoidal I(f) =1.148383
     Absolute error   =0.0005893293
```

3 o

$y' = y + t,\ 0 \le t \le 0.1,\ y(t) = -1 + e^{t} - t$

RK3

```
f[t_, y_] := y + t;
g[t_] := -1 + Exp[t] - t;
a = 0; b = 0.1; n = 1; l = (b - a)/n; t = a; y = g[a];
x2 = N[g[t]]; x3 = Abs[y - x2];
Print["Initial value y0 = ", x2];
Do[Print["STEP  : ", i];
      k1 = f[t, y]; Print["k1 = ", N[k1, 5]];
      k2 = f[t + l/2, y + (l k1)/2]; Print["k2 = ", N[k2, 5]];
      k3 = f[t + l, y - l * k1 + 2 l k2]; Print["k3 = ", N[k3
      x = y + 1/6 l (k1 + 4 k2 + k3);
      t = t + l;
      t1 = N[t];
      x1 = N[x];
      y = x; x2 = N[g[t]]; x3 = Abs[x1 - x2];
      Print["time : ", t1, " num : ", x1, " th : " , x2
```

Initial value y_0 = 0.

STEP : 1

k_1 = 0

k_2 = 0.05

k_3 = 0.11

time : 0.1 num : 0.005166667
  th : 0.005170918 error : 4.251409×10^{-6}