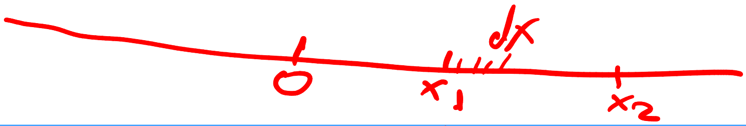


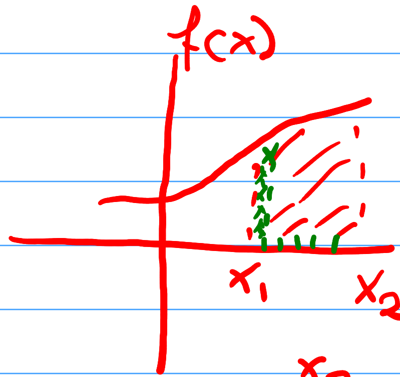
$$a = (6,1 - 1,2 \cdot t) \quad m/s^2$$

$$\alpha = \frac{d\upsilon}{dt} \Rightarrow d\upsilon = \alpha \cdot dt \Rightarrow$$

$$\int_{\upsilon_1}^{\upsilon_2} d\upsilon = \int_{t_1}^{t_2} \alpha \cdot dt \Rightarrow$$

$$\upsilon_2 - \upsilon_1 = \int_{t_1}^{t_2} \alpha \cdot dt \Rightarrow$$

$$\int_{x_1}^{x_2} dx = x \Big|_{x_1}^{x_2} = x_2 - x_1$$



$$\int_{x_1}^{x_2} f(x)dx$$

για $t = 0\ s$

$\upsilon_0 = 2,7\ m/s$

$x_0 = 7,3\ m$

$$* \quad \upsilon - \upsilon_0 = \int_0^t \alpha\, dt \Rightarrow$$

$$\upsilon - 2,7 = \int_0^t (6,1 - 1,2 \cdot t)dt \Rightarrow$$

$$\upsilon - 2,7 = \int_0^t 6,1 \cdot dt - \int_0^t 1,2t \cdot dt \Rightarrow$$

$$\upsilon - 2,7 = 6,1\int_0^t dt - 1,2\int_0^t t\, dt \Rightarrow$$

$$\upsilon - 2,7 = 6,1 \cdot t\Big|_0^t \quad \cancel{-1,2}\frac{t^2}{\cancel{2}}\Big|_0^t$$

$$v - 2,7 = 6,1\,(t-0) - 0,6\left(t^2 - 0^2\right)$$

$$v(t) = -0,6t^2 + 6,1t + 2,7 \qquad (m/s)$$

$$t=0 \rightsquigarrow v(0) = 2,7 \ m/s$$

$$t=6 \rightsquigarrow v(6) = -0,6 \cdot 6^2 + 6,1 \cdot 6 + 2,7$$

$$= \ldots \ m/s$$

$$v = \frac{dx}{dt}$$

$\vdots$

$$v(t) = 0 \Rightarrow -0,6t^2 + 6,1t + 2,7$$

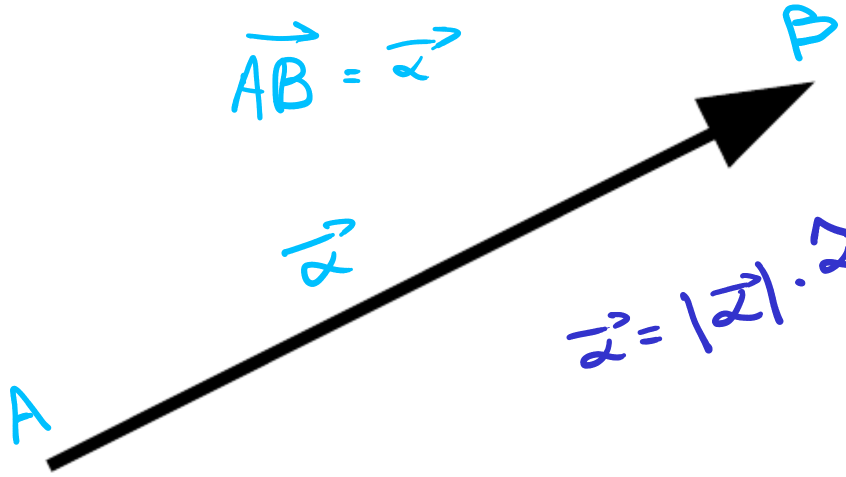
$$\Delta = b^2 - 4ac \Rightarrow \Delta = 6,1^2 - 4(-0,6)\cdot 2,7 \Rightarrow$$

$$\Delta = 37,21 + 6,48 \Rightarrow \Delta = 43,69 > 0$$

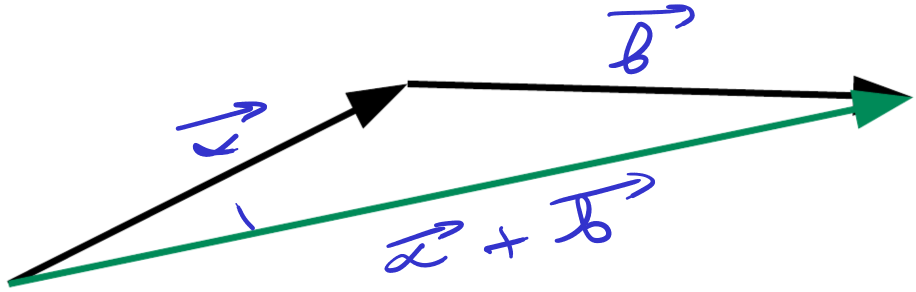
$$t_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} \quad \begin{cases} t_1 = \dfrac{-6,1 + \sqrt{43,69}}{2(-0,6)} \\ t_2 = \dfrac{-}{} \end{cases}$$

$$t_1 = \frac{0,5}{-1,2} \text{ απορ.} \qquad t_2 = \frac{-6,1 - 6,6}{-1,2} = \frac{12,7}{1,2} > 6\,s$$

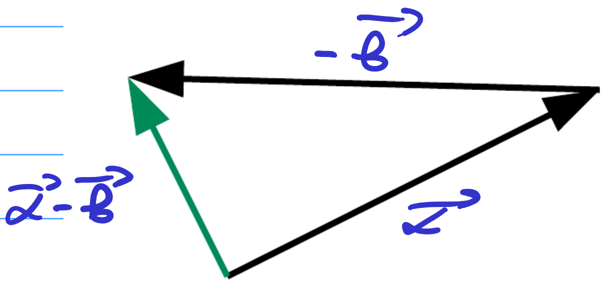
$$\vec{AB} = \vec{\alpha}$$

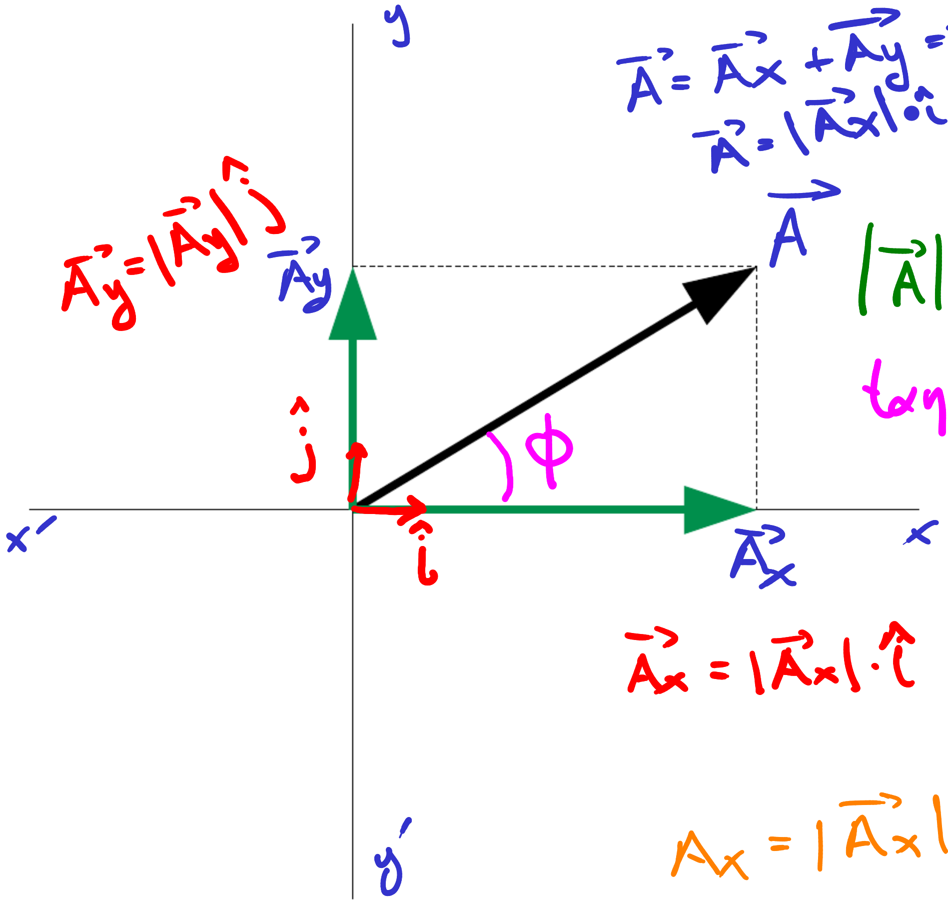


$$\vec{\alpha} = |\vec{\alpha}| \cdot \hat{\alpha}$$



$$\vec{\alpha} - \vec{\beta} = \vec{\alpha} + (-\vec{\beta})$$





$$\vec{A} = \vec{A}_x + \vec{A}_y \Rightarrow \vec{A} = |\vec{A}_x| \cdot \hat{i} + |\vec{A}_y| \hat{j}$$

$$\vec{A}_y = |\vec{A}_y| \hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\tan\phi = \frac{A_y}{A_x}$$

$$\vec{A}_x = |\vec{A}_x| \cdot \hat{i}$$

$$A_x = |\vec{A}_x|$$

Έστω $\vec{A} = A_x \cdot \hat{i} + A_y \hat{j}$

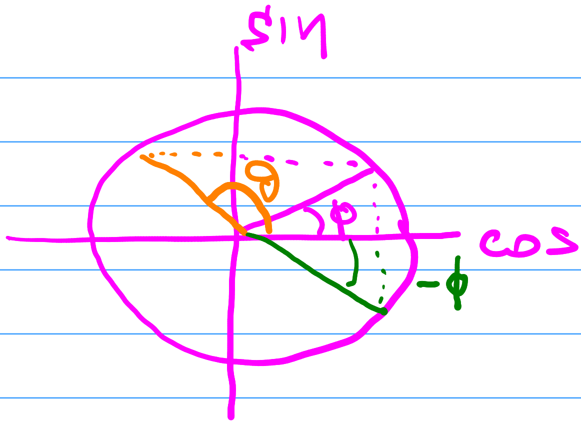
$$\vec{B} = B_x \cdot \hat{i} + B_y \hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

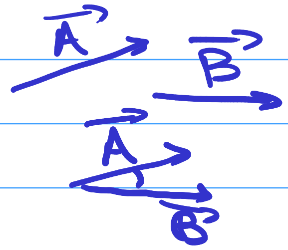
$$\vec{D} = \vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi}$$



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{A} = (A_x, A_y, A_z)$$

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\phi$$



$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot$$

$$\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) =$$

$$A_xB_x\hat{i}\cdot\hat{i} + \cancel{A_xB_y\hat{i}\hat{j}} + \cancel{A_xB_z\hat{i}\cdot\hat{k}}$$

$$+ \cancel{A_yB_x\hat{j}\cdot\hat{i}} + A_yB_y\hat{j}\hat{j} + \cancel{A_yB_z\hat{j}\hat{k}}$$

$$+ \cancel{A_zB_x\hat{k}\hat{i}} + \cancel{A_zB_y\hat{k}\hat{j}} + A_zB_z\hat{k}\cdot\hat{k}$$

$$(\hat{i}\cdot\hat{i} = 1\cdot 1\cdot\cos 0 = 1 = \hat{j}\hat{j} = \hat{k}\hat{k}$$

$$\hat{i}\cdot\hat{j} = 1\cdot 1\cos 90^\circ = 0 = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k})$$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\phi \Rightarrow$$

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$$

Ιδιότητες

$$\vec{\alpha} \cdot \vec{\alpha} \geq 0$$

$$\vec{\alpha} \cdot \vec{\beta} = \vec{\beta} \cdot \vec{\alpha}$$

$$\vec{\alpha}(\vec{\beta} + \vec{\gamma}) = \vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma}$$

$$k(\vec{\alpha} \cdot \vec{\beta}) = (k\vec{\alpha})\vec{\beta} = \vec{\alpha}(k\vec{\beta})$$

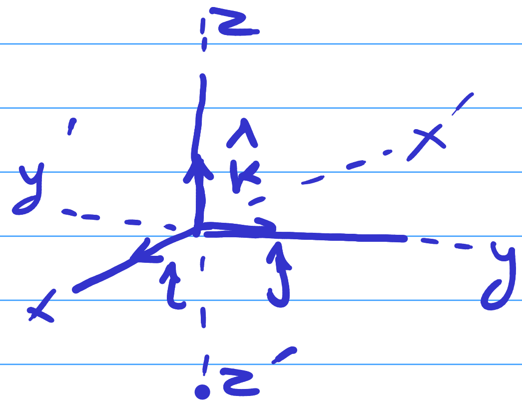
$$\vec{\alpha}^2 = \underline{\vec{\alpha} \cdot \vec{\alpha}} = |\vec{\alpha}| \cdot |\vec{\alpha}| \cos 0$$

$$= \underline{|\vec{\alpha}|^2} =$$

Εξωτερικό γινόμενο

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$



$|\vec{A}\times\vec{B}| = |\vec{A}|\cdot|\vec{B}|\cdot \sin\phi$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$\begin{vmatrix} + & - & + \\ - & & \\ + & & \end{vmatrix}$$

$$\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix}\hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{k}$$

$$= (A_yB_z - B_yA_z)\hat{i}$$

$$- (A_xB_z - B_xA_z)\hat{j}$$

$$+ (A_xB_y - B_xA_y)\hat{k}$$

$$\vec{B}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix} =$$

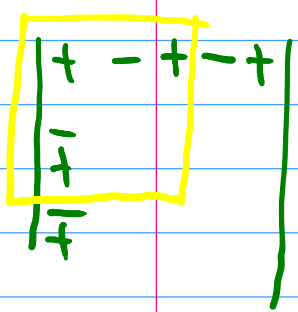
$$= \begin{vmatrix} B_y & B_z \\ A_y & A_z \end{vmatrix}\hat{i} - \begin{vmatrix} B_x & B_z \\ A_x & A_z \end{vmatrix}\hat{j} + \begin{vmatrix} B_x & B_y \\ A_x & A_y \end{vmatrix}\hat{k}$$

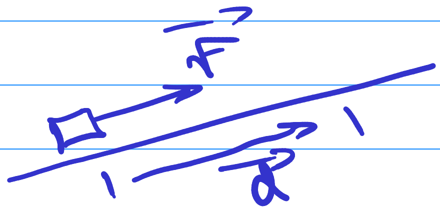
$$= (B_y A_z - A_y B_z)\hat{i}$$
$$- (B_x A_z - A_x B_z)\hat{j}$$
$$+ (B_x A_y - A_x B_y)\hat{k}$$

$$\vec{A} = 8\hat{i} - 2\hat{j} + 3\hat{k}$$
$$\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$$

$\vec{A} \times \vec{B}$, $\vec{B} \times \vec{A}$

$$\vec{A} \times \vec{B} = -7\hat{i} + 20\hat{j} + 32\hat{k}$$
$$\vec{B} \times \vec{A} = 7\hat{i} - 20\hat{j} - 32\hat{k}$$

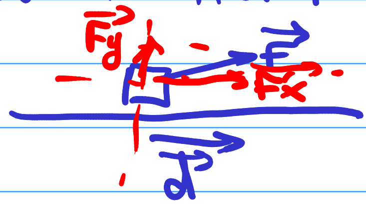




$$\vec{F} = 2\hat{i} + 5\hat{j} + 4\hat{k} \ (N)$$
$$\vec{d} = 4\hat{i} + 4\hat{j} + 4\hat{k} \ (m)$$

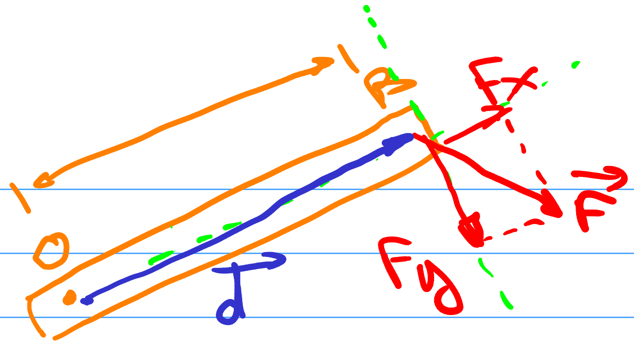
$$W = |\vec{F}||\vec{d}| \cdot \cos\phi = \vec{F} \cdot \vec{d}$$

$W =$;



$$W = \cancel{|\vec{F}| \cdot |\vec{d}|} \cdot \frac{\vec{F} \cdot \vec{d}}{\cancel{|\vec{F}| \cdot |\vec{d}|}}$$

$$W = 8 + 20 + 16 = 44 \ (J)$$



$\tau = F \cdot d \cdot \sin\phi$

$\vec{\tau} = \vec{d} \times \vec{F}$

$\vec{d} = 3\hat{i} + 2\hat{j}$

$\vec{F} = 5\hat{i} - 4\hat{j} - 2\hat{k}$

$\vec{\tau} = ;$

$\vec{\tau} = 4\hat{i} - 22\hat{k}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 5 & -4 & -2 \end{vmatrix} = (-4-(-4)\cdot 0)\hat{i} - (-6)\hat{j} + (-12-10)\hat{k} = -4\hat{i} + 6\hat{j} - 22\hat{k} \ (N.m)$$

$$|\vec{\tau}| = \sqrt{(-4)^2 + 6^2 + (-22)^2}$$

$$= \sqrt{16 + 36 + 484}$$

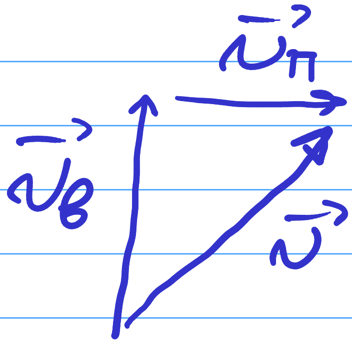
$$= \sqrt{} = 23{,}15 \ N.m$$

$\sin\phi = ;$ $\qquad \cos\phi = \dfrac{\vec{F} \cdot \vec{d}}{|\vec{F}| \cdot |\vec{d}|}$

$$\cos\phi = \frac{7}{\sqrt{45} \cdot \sqrt{13}}$$

$$\sin^2\phi + \cos^2\phi = 1 \Rightarrow \sin\phi = \pm\sqrt{1 - \cos^2\phi}$$

$$\vec{v} = \underline{\underline{\vec{v}_B}} + \underline{\underline{\vec{v}_\Pi}}$$



$\vec{\alpha}\cdot\vec{\beta}$

$\vec{\alpha}\times\vec{\beta}$

$\vec{\alpha}\cdot(\vec{\beta}\cdot\vec{\gamma})$

$\vec{\gamma}(\vec{\alpha}\times\vec{\beta})$

$$\vec{\alpha} = \alpha_1\hat{i} + \alpha_2\hat{j} + \alpha_3\hat{k}$$

$$\vec{\beta} = \beta_1\hat{i} + \beta_2\hat{j} + \beta_3\hat{k}$$

$$\vec{\gamma} = \gamma_1\hat{i} + \gamma_2\hat{j} + \gamma_3\hat{k}$$

$$\vec{\alpha} \times \vec{\beta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}$$

$$\vec{\gamma}(\vec{\alpha} \times \vec{\beta}) = (\gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}$$

$$= \begin{vmatrix} \gamma_1 & \gamma_2 & \gamma_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}$$

$\vec{\alpha} = 4\hat{i} - 3\hat{j} + 1\hat{k}$

$\vec{\beta} = -1\hat{i} + 1\hat{j} + 4\hat{k}$

α) $\vec{\alpha} + \vec{\beta}$ β) $\vec{\alpha} - \vec{\beta}$

γ) $\vec{c}$; : $\vec{\alpha} - \vec{\beta} + \vec{c} = 0$

δ) $\vec{\alpha} \cdot \vec{\beta}$ ε) $\vec{\alpha} \times \vec{\beta}$

ζ) $\vec{c}(\vec{\alpha} \times \vec{\beta})$