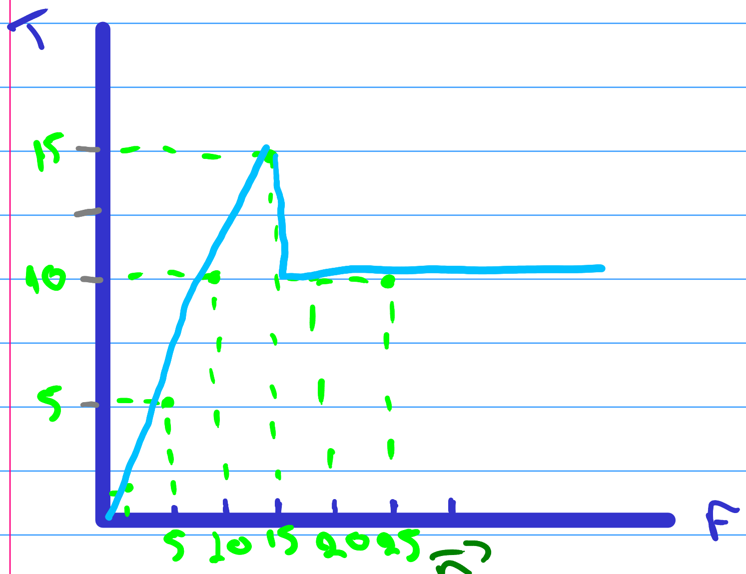
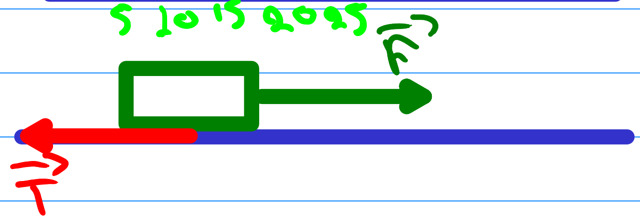




$m = 5\ kg$
$g = 10\ m/s^2$

$\mu_{στ} = 0{,}30$
$\mu_{ολ} = 0{,}20$

$T = \mu N$
$N = w = mg$
$N = 5 \cdot 10 = 50N$

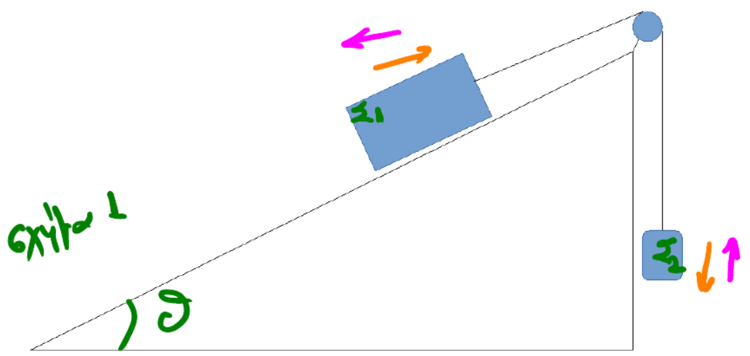
F(N)	0	1	5	10	15	20	25	30	
T(N)	0	1	5	10	15	10	10	10	

$T_{στ} \leq \mu_{στ} \cdot N \Rightarrow T_{στ} \leq 0{,}30 \cdot 50 \Rightarrow$

$T_{στ} \leq 15N$

$T_{ολ} = \mu_{ολ} \cdot N \Rightarrow T_{ολ} = 0{,}20 \cdot 50 \Rightarrow T_{ολ} = 10N$

3)



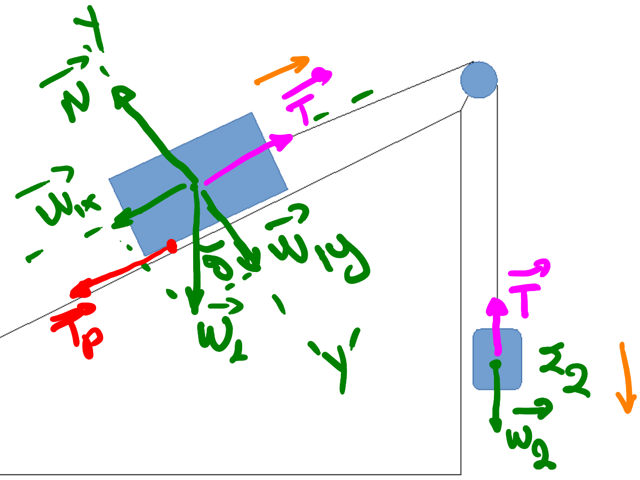
Γνωστά: $m_1, m_2, g, \theta, \mu_{στ}$

Σώματα ακίνητα

α)

β)

Σχέση $m_1, m_2, g, \theta, \mu_{στ}$



$Σ_2$: $\Sigma F = 0 \Rightarrow T - w_2 = 0 \Rightarrow T = w_2 \Rightarrow T = m_2 g$ (1)

$Σ_1$: $\Sigma F = 0 \Rightarrow \Sigma F_y = 0 \Rightarrow N - w_{1y} = 0 \Rightarrow$

$N = w_{1y} \Rightarrow N = m_1 g \cos\theta$ (2)

$\Sigma F_x = 0 \Rightarrow T - w_{1x} - T_p = 0 \Rightarrow$

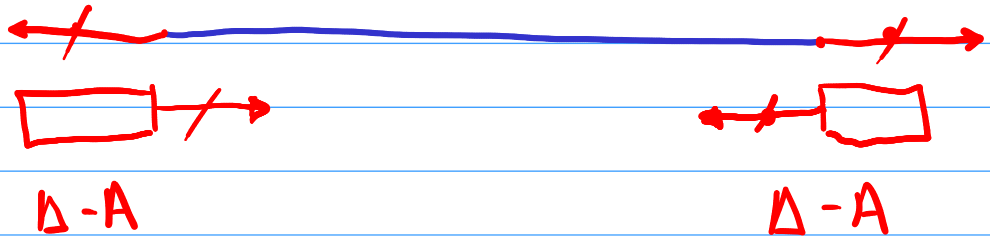
$T = w_{1x} + T_p \Rightarrow T = m_1 g \sin\theta + \mu_{στ} N$ (3)

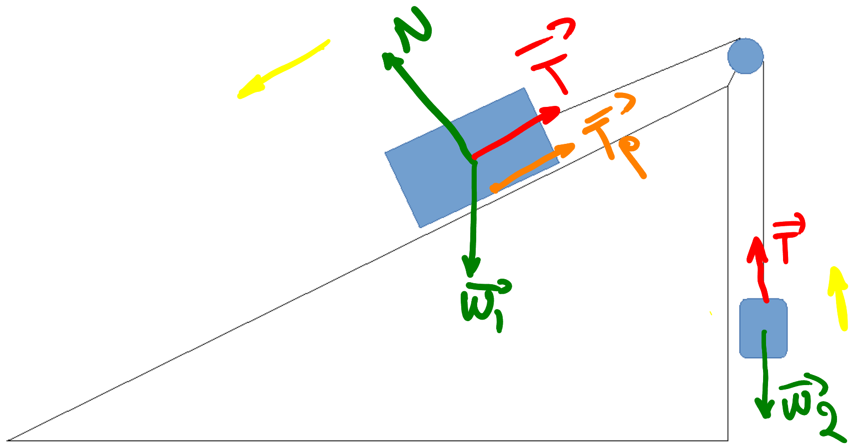
③ ② $\Rightarrow$ $T = m_1 g \sin\theta + \mu_{sz} \cdot m_1 g \cdot \cos\theta$

Ⅰ $\Rightarrow$ $T = m_2 \cdot g$

$\Rightarrow$

$$m_2 g = m_1 g \sin\theta + \mu_{sz} m_1 g \cos\theta$$





Σ_2: $T = W_2$

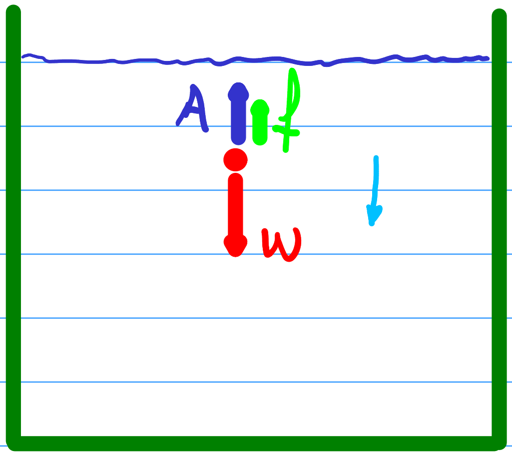
Σ_1: $\Sigma F_x =$

$T + T_p - W_{1x} = 0$

ΣF_y : $N = W_{1y}$

⋮

Αντίσταση ρευστού & Ορική ταχύτητα



Το κινούμενο σώμα ασκεί δύναμη στο ρευστό για να το βγάλει από τη διαδρομή του.
Το ρευστό σπρώχνει με ίση και αντίθετη δύναμη το σώμα (3ος νόμος)

Δύναμη αντίστασης υγρού (ρευστού) → $f = k\upsilon$ *

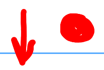
→ $f = k\upsilon^2$ **

* αντίσταση για μικρές ταχύτητες

** αντίσταση για μεγάλες ταχύτητες

Πτώση πέτρας σε λίμνη

$f = k \cdot \upsilon$



$\Sigma F_y = m \cdot a_y \Rightarrow$

$mg - k\upsilon_y = m \cdot a_y$ ↙ ορική ταχύτητα

για $a_y = 0$: $mg - k\upsilon_{op} = 0 \Rightarrow \upsilon_{op} = \frac{mg}{k}$

Άνωση: $A = \varepsilon \cdot V$ $\quad \varepsilon = \rho \cdot g$

Για διαγράμματα $\alpha - t$, $v - t$, $x - t$, $y - t$

$\Sigma F = m\alpha_y \Rightarrow$

$$mg - kv_y = m\frac{dv_y}{dt} \Rightarrow$$

$v_{op} = \frac{mg}{k}$

$$\frac{mg}{k} - \frac{kv_y}{k} = \frac{m}{k}\cdot\frac{dv_y}{dt} \Rightarrow$$

↓

$$v_{op} - v_y = \frac{m}{k}\frac{dv_y}{dt} \Rightarrow$$

$$(v_{op} - v_y)\cdot dt = \frac{m}{k} dv_y \Rightarrow \frac{k}{m} dt = \frac{1}{(v_{op} - v_y)} dv_y$$

$t = 0$, $v = 0$

$$\int_0^t \frac{k}{m} dt = \int_0^v \frac{1}{(v_{op} - v_y)} dv_y \quad (1)$$

Θέτω $z = v_{op} - v_y$ $\quad z' = v_{op}' - v_y'$

$$dz = 0 - dv_y$$

$$dv_y = -d(v_{op} - v_y)$$

$$dz = d(v_{op} - v_y) = -dv_y$$

(1) $$\frac{k}{m}\cdot t = -\int_0^v \frac{1}{v_{op} - v_y} d(v_{op} - v_y) \Rightarrow$$

$$-\frac{k}{m} t = \ln|v_{op} - v_y| \Big|_0^v \Rightarrow -\frac{k}{m} t = \ln|v_{op} - v| - \ln|v_{op} - 0|$$

$$\Rightarrow -\frac{k}{m}t = \ln(\upsilon_{op} - \upsilon) - \ln \upsilon_{op} \Rightarrow$$

$$-\frac{k}{m}t = \ln \frac{\upsilon_{op} - \upsilon}{\upsilon_{op}} \Rightarrow$$

$$e^{-\frac{k}{m}t} = \frac{\upsilon_{op} - \upsilon}{\upsilon_{op}} \Rightarrow e^{-\frac{k}{m}t} = 1 - \frac{\upsilon}{\upsilon_{op}} \Rightarrow$$

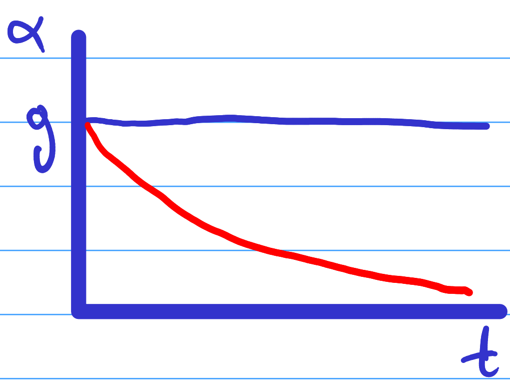
$$\Rightarrow \frac{\upsilon}{\upsilon_{op}} = 1 - e^{-\frac{k}{m}t} \Rightarrow \upsilon = \upsilon_{op}\left(1 - e^{-\frac{k}{m}t}\right)$$

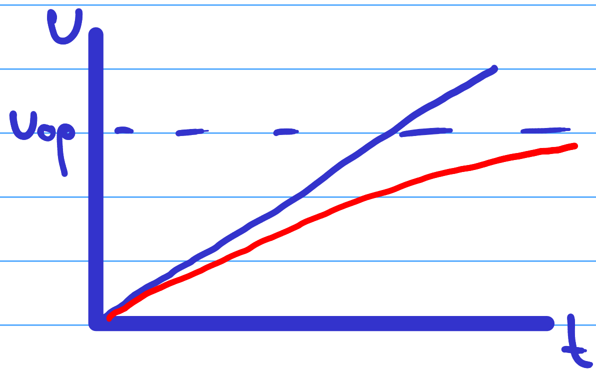
γιά $t \to \infty$
$\upsilon \to \upsilon_{op}$

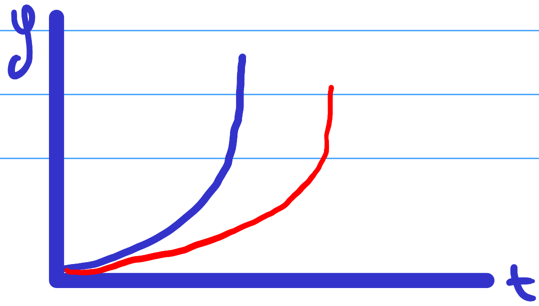
$$\boxed{\upsilon = \frac{m \cdot g}{k}\left(1 - e^{-\frac{k}{m}t}\right)}$$

$$\alpha = \frac{d\upsilon}{dt} \Rightarrow \alpha = g\, e^{-\frac{k}{m}t}$$

$$y = \int_0^y dy = \int_0^t \upsilon\, dt \Rightarrow y = \upsilon_{op}\left[t - \frac{m}{k}\left(1 - e^{-\frac{k}{m}t}\right)\right]$$







Για μεγάλες ταχύτητες (π.χ. πτώση στον αέρα)

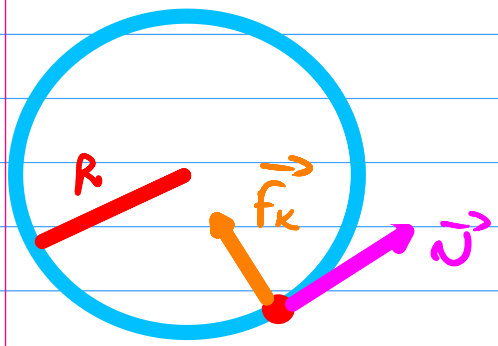
$f = k v^2$

$$\Sigma F = ma \Rightarrow$$

$$mg - k v^2 = ma$$

για $a = 0$: $mg - kv^2 = 0 \Rightarrow v = \sqrt{\frac{m \cdot g}{k}}$

Δυναμική της κυκλικής κίνησης



$$\alpha_{κεντ} = \frac{v^2}{R}$$

$$F_{κεντρ} = m\,\alpha_{κεντ}$$

$$F_{κεντρ} = m\,\frac{v^2}{R}$$

$$v = \frac{\Delta s}{\Delta t} \Rightarrow v = \frac{2\pi R}{T}$$

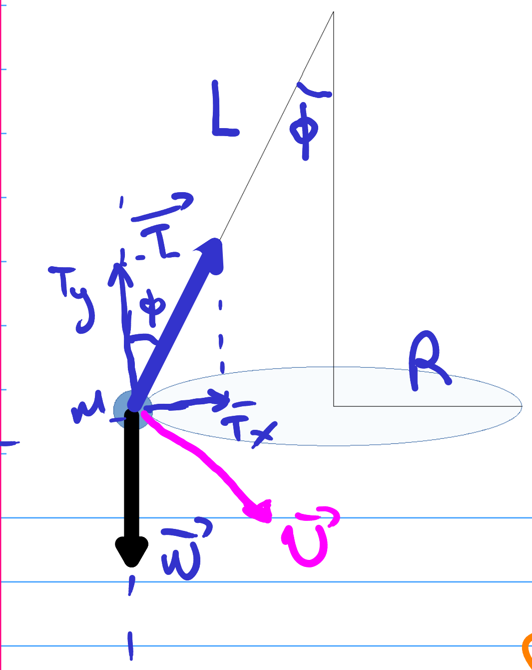
$$T = \frac{2\pi R}{v}$$

$$\alpha_{κεντ} = \frac{4\pi^2 R}{T^2}$$

Πχ $m = 30{,}0\,kg$, $R = 3{,}00\,m$, $10\ \frac{περιστροφές}{min}$

$\alpha_K = ;$ $F_K = ;$

Για κωνικό εκκρεμές



γνωστά L, ϕ, m, g

$T_y = w \Rightarrow T \cdot \cos\phi = mg$ (1)

T_x = Fκεντρομόλος

$T_x = m a_\kappa \Rightarrow$

$T \sin\phi = m a_\kappa$ (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{T \sin\phi}{T \cos\phi} = \frac{m a_\kappa}{mg} \Rightarrow$$

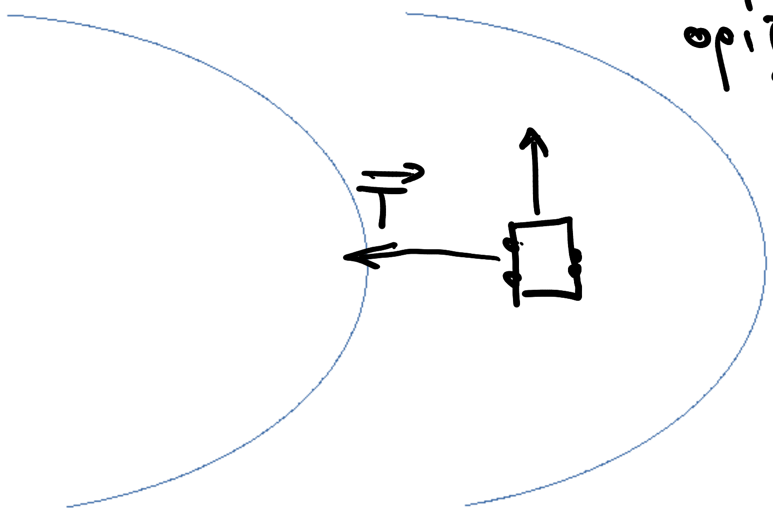
$$\tan\phi = \frac{a_\kappa}{g} \Rightarrow$$

$$\tan\phi = \frac{\frac{4\pi^2 R}{T^2}}{g} \Rightarrow T^2 = \frac{4\pi^2 R}{g \tan\phi} \Rightarrow$$

$R = L \cdot \sin\phi$

$$T = 2\pi \sqrt{\frac{L \cos\phi}{g}}$$

κίνηση σε οριζόντια στροφή



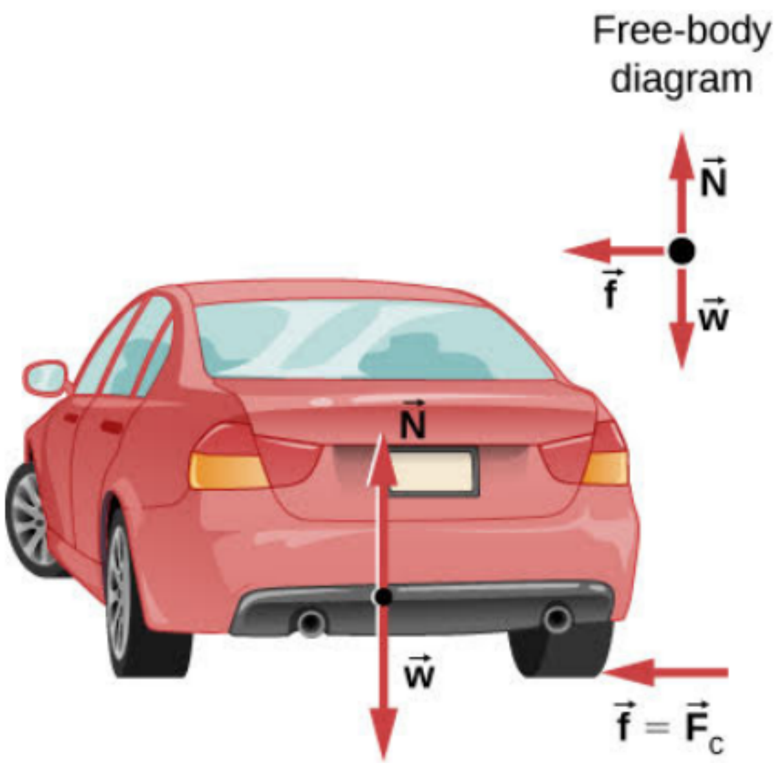
$N = w \Rightarrow$

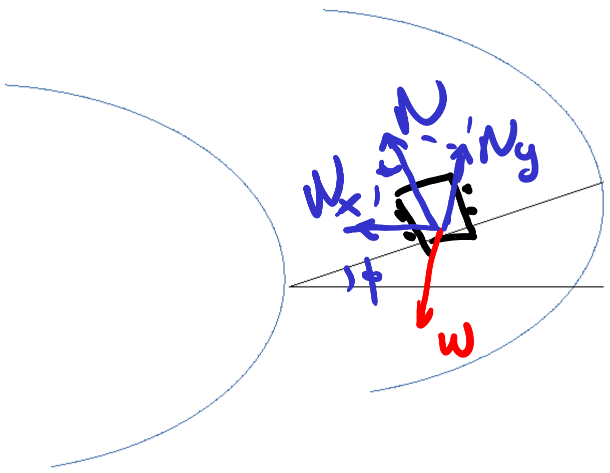
$N = mg$

$$T = m\alpha_\kappa \Rightarrow \mu \cdot N = m\frac{v^2}{R}$$

$$\mu m g = m\frac{v^2}{R} \Rightarrow$$

$$v = \sqrt{\mu \cdot g \cdot R}$$





$$N \cdot \sin\phi = m a_k$$

$$N \cdot \cos\phi = mg$$

$$\tan\phi = \frac{a_k}{g}$$

$$a_k = g \tan\phi$$

$$\frac{v^2}{R} \Rightarrow v = \sqrt{R g \tan\phi}$$

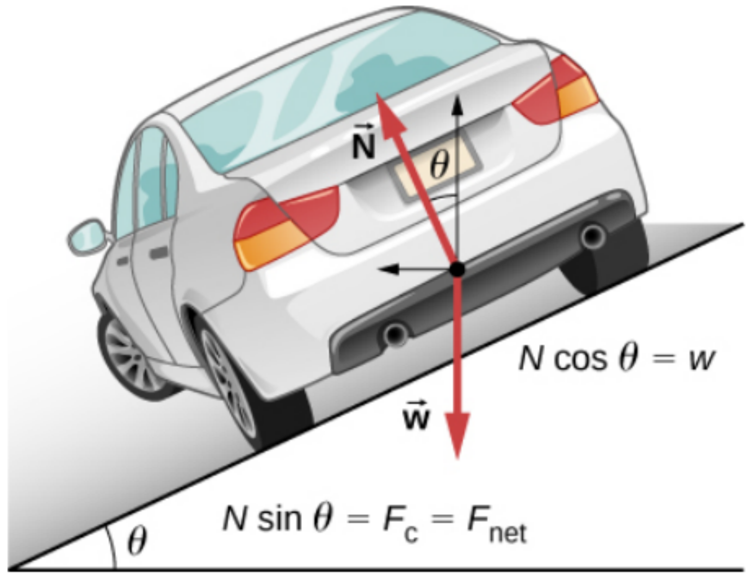
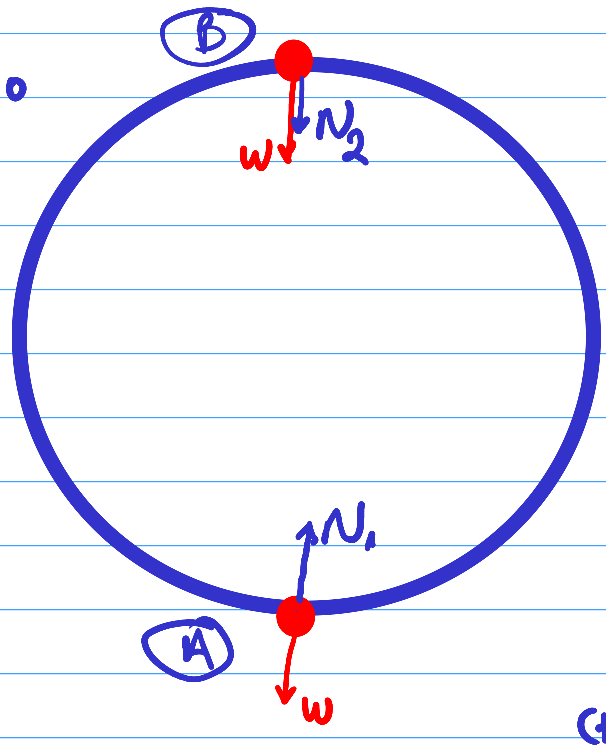


Figure 6.22 The car on this banked curve is moving away and turning to the left.

κίνηση σε κατακόρυφο κύκλο



A : $\Sigma F_y = m \cdot \alpha_\kappa$

$N_1 - mg = m \cdot \alpha_\kappa$

B: $\Sigma F_y = m\alpha_\kappa$

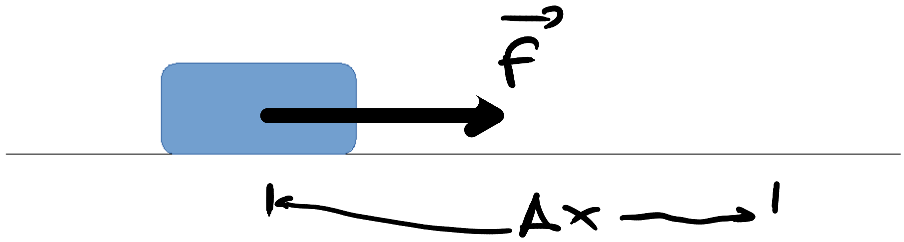
(+)↓ $N_2 + w = m\alpha_\kappa$

(+)↑ $\Sigma F_y = -m\alpha_\kappa$

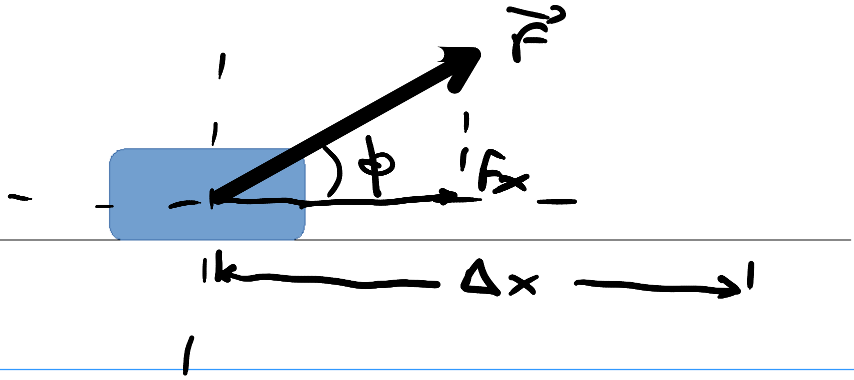
$-N - w = -m\alpha_\kappa$

Έργο

$W = \vec{F} \cdot \vec{s}$ $\qquad (W = F \cdot s \cdot \cos\theta)$



$W = F \cdot \Delta x$



$W = F_x \cdot \Delta x$

$W = F \cdot \Delta x \cos\phi$

Π.χ. έστω $\vec{F} = (80N)\hat{i} + (50N)\hat{j}$

$\vec{s} = \Delta\vec{x} = (10m)\hat{i} + (25m)\hat{j}$

$W = \vec{F} \cdot \vec{s} \Rightarrow W = 80 \cdot 10\, N \cdot m + 50 \cdot 25\, N \cdot m$

$\Rightarrow W = 800\, N \cdot m + 1250\, N \cdot m$

$\Rightarrow W = 2050$ Joule.

Έργο από πολλές δυνάμεις

$W = \Sigma\vec{F} \cdot \Delta\vec{x}$ ή $W = \vec{F_1} \cdot \Delta\vec{x} + \vec{F_2} \cdot \Delta x + \ldots$

αν $\Delta\vec{x} = 0$ τότε $W = 0$

αν $\Sigma\vec{F} = 0$ και $\Delta\vec{x} \neq 0 \Rightarrow W = 0$

↳ $\vec{\upsilon} =$ σταθερό

ΘΜΚΕ : $W_{ολ} = \Delta K = K_2 - K_1$

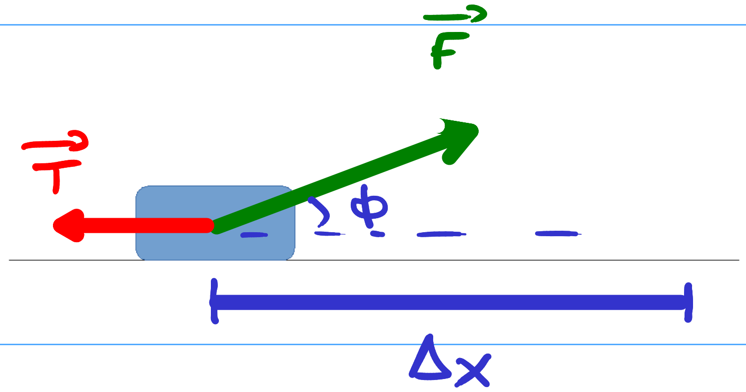
Το έργο που παράγεται από τη συνισταμένη δύναμη πάνω σε ένα σωμάτιο είναι ίσο με τη μεταβολή της κινητικής ενέργειας του σωματίου.

$\left(K = \frac{1}{2} m \upsilon^2\right)$

$\upsilon^2 = \vec{\upsilon}^2$

Μονάδες έργου Joule

$1J = 1N \cdot m = 1\,kg\frac{m}{s^2} \cdot m = 1\,kg\frac{m^2}{s^2}$



$F = 20\sqrt{3}N$

$w = 10N$

$\mu_{o\lambda} = 0{,}2$

$\phi = 30^\circ$

$s = \Delta x = 20m$

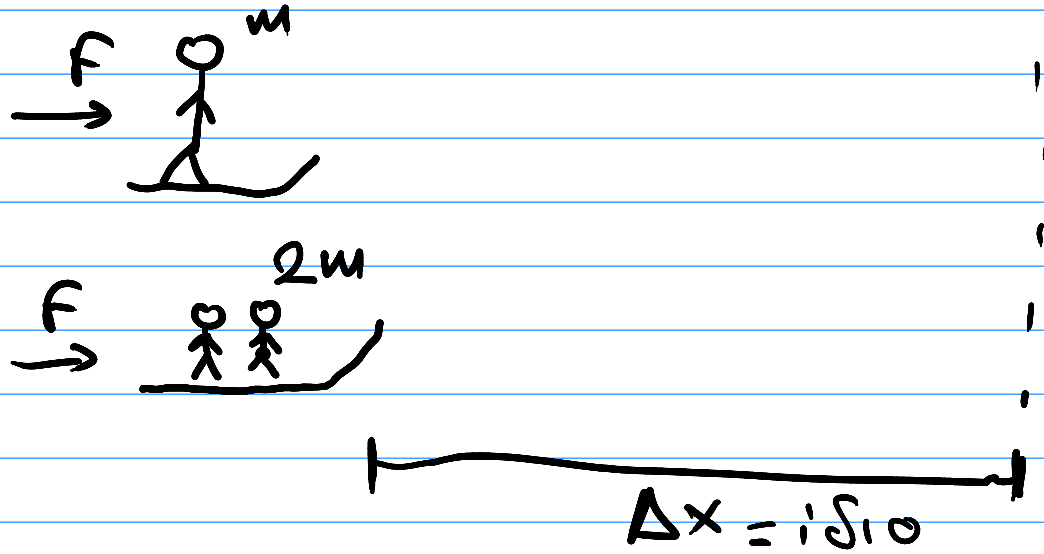
$\upsilon_0 = 3m/s$

$\upsilon_{\tau\varepsilon\lambda\iota\kappa\acute{\eta}} = ;$

α) εξ. κίνησης

β) ΘΜΚΕ

ίδια F



$K_{τελ} = ;$

$υ_{τελ} = ;$