$\triangleright \xrightarrow{v_x}$ $F - kv_x^2 = \frac{dv_x}{dt} \cdot m$

$$\int F\,dt + \int -\frac{k}{m}\,dt = \int \frac{1}{v_x^2}\,dx$$

$-\ \vec{v} = 0$

$\vec{g}$  $\uparrow \vec{f} = kv_y^2$ $\qquad \Sigma F_y = m \cdot a_y$

$\vec{v}$ $\downarrow \vec{w} = mg$

$$mg - kv_y^2 = m\frac{dv_y}{dt}$$

$$\frac{mg}{k} - v_y^2 = \frac{m}{k}\frac{dv_y}{dt} \quad *$$

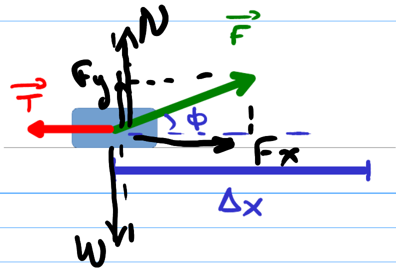
$-\ \vec{v}_{op}$

$\hookrightarrow$ $\Sigma F_y = 0 \Rightarrow mg - kv_{yop}^2 = 0 \Rightarrow v_{yop} = \sqrt{\frac{mg}{k}}$

$$* \quad v_{yop}^2 - v_y^2 = \frac{m}{k}\frac{dv_y}{dt} \Rightarrow$$

$$\int_0^t \frac{k}{m}\,dt = \int_0^v \frac{1}{v_{yop}^2 - v_y^2}\,dv_y$$

(+) →



$F = 20\sqrt{3}\,N$

$w = 10\,N$

$\mu_{ολ} = 0,2$

$\varphi = 30^\circ$

$s = \Delta x = 20\,m$

$\upsilon_0 = 3\,m/s$

$\upsilon_{τελική} = ;$

~~$g = 9,8\,m/s^2$~~

$g = 10\,m/s^2$

α) εξ. κίνησης

β) ΘΜΚΕ

$T = \mu \cdot N$

$\Sigma F_y = 0 \Rightarrow N + F_y - w = 0$

$N = w - F_y \Rightarrow$

$N = mg - F\sin\varphi$

α) $\Sigma F_x = m\alpha_x \Rightarrow$

$F_x - T = m\alpha_x \Rightarrow$

$F\cdot\cos\varphi - \mu\cdot N = m\alpha_x$

$$\alpha_x = \frac{F\cos\varphi - \mu N}{m}$$

(α_x = σταθερή)

$\upsilon_x = \upsilon_{0x} + \alpha t \quad ;$

$$\Delta x = \upsilon_{0x} t + \frac{1}{2}\alpha_x t^2 \rightarrow \alpha_x t^2 + 2\upsilon_{0x} t - \Delta x = 0$$

$$\Delta = \beta^2 - 4\alpha\gamma \Rightarrow \Delta = 4\upsilon_{0x}^2 - 4\cdot\alpha_x(-\Delta x)$$

$$t_1 = \frac{-2\upsilon_{0x} + \sqrt{\Delta}}{2\alpha_x}$$

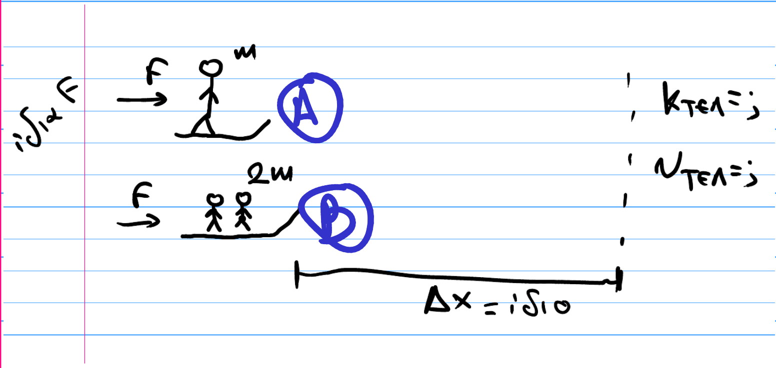
~~$$t_2 = \frac{-2\upsilon_{0x} - \sqrt{\Delta}}{2\alpha_x}$$~~

β) $\Delta K = W_{\Sigma F_x} \Rightarrow K_2 - K_1 = \Sigma F_x \cdot \Delta x \Rightarrow$

$$\frac{1}{2}m\upsilon^2 - \frac{1}{2}m\upsilon_0^2 = (F_x - T)\Delta x \Rightarrow$$

↑

;



$K_{αρχ} = 0$

$\Delta K = W_{\Sigma F}$

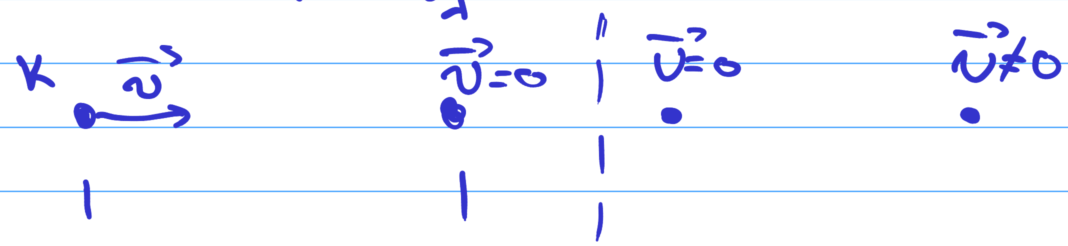
$K_2 - K_1 = F \cdot \Delta x$

$K_2 = F \cdot \Delta x$ (ίδια)

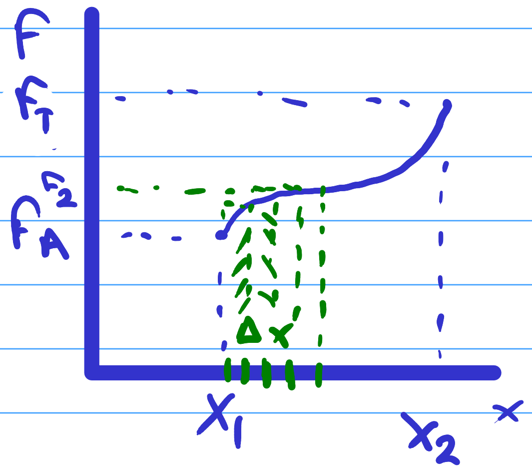
(A) $K_2 = \frac{1}{2} m \upsilon_A^2 \Rightarrow F \cdot \Delta x = \frac{1}{2} m \upsilon_A^2 \Rightarrow \upsilon_A = \sqrt{\frac{2F \cdot \Delta x}{m}}$

(B) $K_2 = \frac{1}{2} 2m \upsilon_B^2 \Rightarrow \; . \; . \; . \; . \; . \Rightarrow \upsilon_B = \sqrt{\frac{F \cdot \Delta x}{m}}$

$K_{αρχικό} = 0$ $\qquad K = W_{ολ}$ $\qquad K$



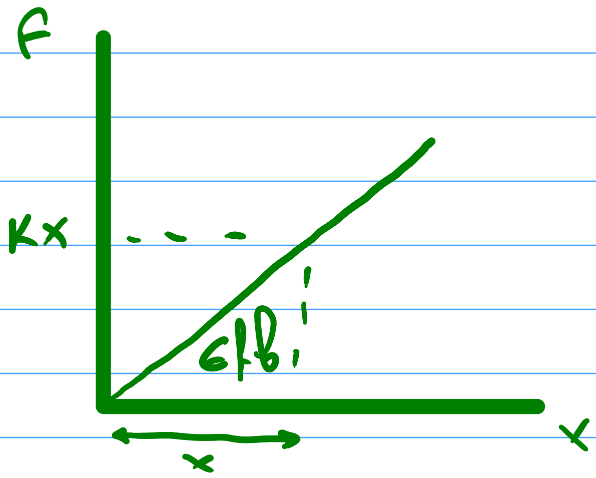
Μεταβαλλόμενες Δυνάμεις



W_F ;

$$W_{(\Delta x)} = F_1 \cdot \Delta x + F_2 \cdot \Delta x + \dots$$

$$W_{x_1 \to x_2} = \int_{x_1}^{x_2} F \cdot dx$$



Δύναμη (πάνω) ελατηρίου

$$Εμβ = \frac{β \cdot υ}{2}$$

$$Εμβ = \frac{Kx \cdot x}{2}$$

$$Εμβ = \frac{1}{2} K x^2$$

$$w = \int_0^x F \cdot dx \Rightarrow w = \int_0^x Kx\,dx \Rightarrow w = K \int_0^x x\,dx$$

$$\Rightarrow w = K\left(\frac{x^2}{2}\right)\Big|_0^x \Rightarrow$$

$$\Rightarrow w = \frac{1}{2} K x^2$$

Τρεις διαστάσεις

$$dW = \vec{F}\cdot d\vec{r} \Rightarrow dW = F_x dx + F_y dy + F_z dz$$

$$\vec{F} = F_x\cdot\hat{i} + F_y\cdot\hat{j} + F_z\cdot\hat{k}$$

$$d\vec{r} = dx\cdot\hat{i} + dy\cdot\hat{j} + dz\cdot\hat{k}$$

$$W = \int_{W_1}^{W_2} dW = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z\cdot dz$$

$W_{x_1\to x_2}$

$$W = \int_{x_1}^{x_2} F dx \Rightarrow W = \int_{x_1}^{x_2} m\cdot a\cdot dx \quad (1) \qquad (F = m\cdot a)$$

$madx$;

$$a = \frac{du}{dt} \Rightarrow a = \frac{du}{dx}\cdot\frac{dx}{dt} \qquad \left|\ \frac{\alpha}{\beta}\cdot\frac{\gamma}{\gamma} = \frac{\alpha\gamma}{\gamma\beta}\right.$$

$$a = \frac{du}{dx}\cdot u$$

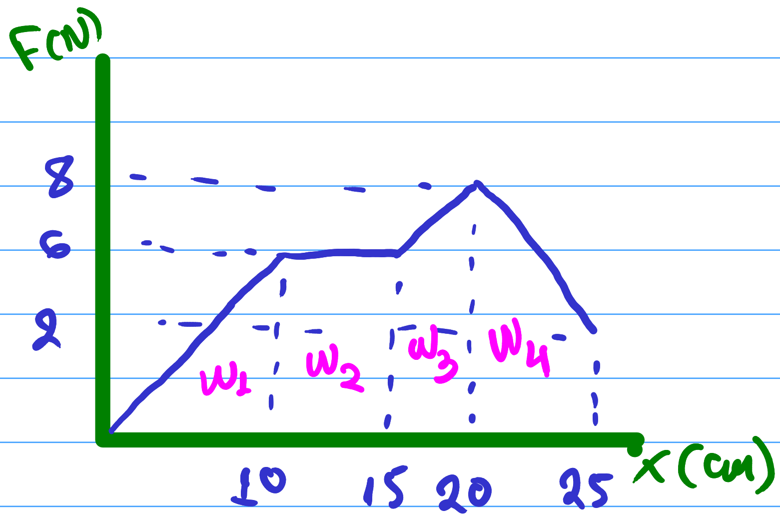
$$m\,a\,dx = m\cdot\frac{du}{dx}\cdot u\cdot dx \Rightarrow m\,a\,dx = m\,u\,du$$

$$(1) \Rightarrow W = \int_{u_1}^{u_2} m\,u\,du \Rightarrow W = m\int_{u_1}^{u_2} u\,du \Rightarrow$$

$$\Rightarrow W = m\left(\frac{u^2}{2}\right)\Bigg|_{u_1}^{u_2} \Rightarrow$$

$$W = \frac{1}{2}mu_2^2 - \frac{1}{2}mu_1^2 \Rightarrow W = K_2 - K_1$$

$$K = \frac{1}{2}mu^2$$



$W_{o\lambda} = ;$

$W_1 = \frac{1}{2} 10 \cdot 6 \cdot 10^{-2} \Rightarrow$

$W_1 = 30 \cdot 10^{-2}\ J$

$W_2 = (15-10) \cdot 10^{-2} \cdot 6$

$W_2 = 30 \cdot 10^{-2}\ J$

$W_3 = \frac{(8+6) \cdot (20-15) \cdot 10^{-2}}{2} \Rightarrow$

$W_3 = 35 \cdot 10^{-2}\ J$

$W_4 = \frac{(8+2) \cdot (25-20) \cdot 10^{-2}}{2} \Rightarrow W_4 = 25 \cdot 10^{-2}\ J$

$W_{o\lambda} = 120 \cdot 10^{-2}\ J$

$\vec{F} = (3x^2\,N)\hat{i} + (4N)\hat{j}$

$A(2m, 3m) \longrightarrow B(3m, 0)$

$W = ;$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \Rightarrow W = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy$$

$$\Rightarrow W = \int_2^3 3x^2\,dx + \int_3^0 4 \cdot dy \Rightarrow$$

$3\int x^2 d \rightarrow 3\frac{x^3}{3}$

$$W = 3\frac{x^3}{3}\Big|_2^3 + 4y\Big|_3^0 \Rightarrow$$

$$W = (3^3 - 2^3) + 4 \cdot (0 - 3) \Rightarrow$$

$$W = 19 - 12 \Rightarrow W = 7\,J$$

Ισχύς

$\bar{P} = P_\mu = \dfrac{W}{\Delta t}$

στιγμιαία Ισχύς $\Rightarrow P = \dfrac{dW}{dt}$

$P \rightarrow$ Watt $\quad 1\,\text{Watt} = 1\,\dfrac{\text{Joule}}{s}$

$P \rightarrow$ hp $\quad 1\,\text{hp} = 746\,\text{watt} = 0{,}746\,\text{kW}$

Ενέργεια $\rightarrow$ Joule

$\hookrightarrow$ $1\,\text{kWh} = 1\,\text{kW} \cdot 1\,\text{h} = 1000\,\text{W} \cdot 3600\,s$

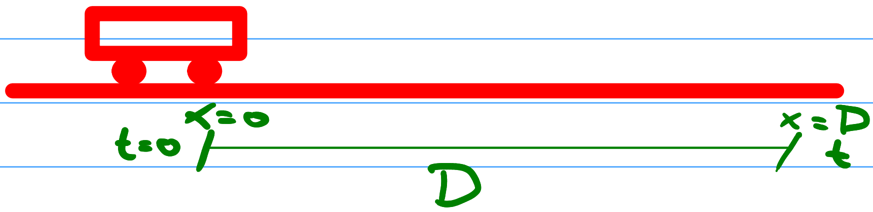
$\Rightarrow 1\,\text{kWh} = 3600000\,\text{J} = 3{,}6\,\text{MJ}$

$P = F \cdot v$

$P = \dfrac{dW}{dt} \Rightarrow P = \dfrac{\vec{F} \cdot d\vec{x}}{dt} \Rightarrow$

$P = \dfrac{F \cos\phi \, dx}{dt} \Rightarrow$

$P = F \cdot \cos\phi \cdot v \Rightarrow P = \vec{F} \cdot \vec{v}$



P, m, D

$t = ;$

$P = \frac{dw}{dt}$ $\qquad dw = dk$

$P = \frac{dk}{dt} \Rightarrow dk = P \cdot dt \Rightarrow \int_0^k dk = \int_0^t P\,dt \Rightarrow$

$\Rightarrow k = P \cdot t \Rightarrow \frac{1}{2} m v^2 = P \cdot t \Rightarrow$

$v = \sqrt{\frac{2P \cdot t}{m}} \Rightarrow v = \left(\frac{2P \cdot t}{m}\right)^{1/2}$

$v = \frac{dx}{dt} \Rightarrow dx = v \cdot dt \Rightarrow \int_0^D dx = \int_0^t v\,dt \Rightarrow$

$$D = \int_0^t \left(\frac{2Pt}{m}\right)^{1/2} dt \Rightarrow$$

$$D = \left(\frac{2P}{m}\right)^{1/2} \cdot \int_0^t t^{1/2}\,dt \Rightarrow$$

$$D = \left(\frac{2P}{m}\right)^{1/2} \frac{t^{3/2}}{\frac{3}{2}} \Rightarrow$$

$$\frac{3}{2} D = \left(\frac{2P}{m}\right)^{1/2} \cdot t^{3/2} \Rightarrow$$

$$t = \left[\frac{3}{2} D \cdot \left(\frac{m}{2P}\right)^{1/2}\right]^{2/3} \Rightarrow$$

$$t = \left(\frac{3}{2} D\right)^{2/3} \cdot \left(\frac{m}{2P}\right)^{1/3}$$

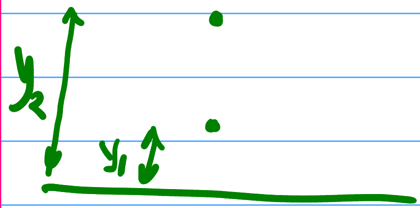
Έργο Ελατηρίου

$$F = -kd \quad , \quad F = -kx$$

$$W = \int_{x_1}^{x_2} (-F_x\, dx) \Rightarrow W = -\int_{x_1}^{x_2} kx\, dx \Rightarrow$$

$$W = -\frac{1}{2}k\left(x_2^2 - x_1^2\right) \Rightarrow W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Βαρυτική δυναμική ενέργεια



$$W = mg(y_1 - y_2)$$

→ (+) $y_1 > y_2$

→ (−) $y_1 < y_2$

↓

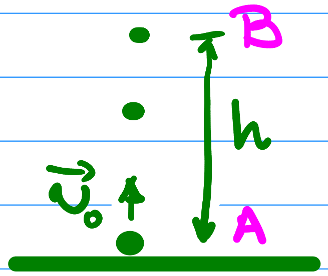
$$W = mgy_1 - mgy_2 \Rightarrow$$

$$W = -(mgy_2 - mgy_1)$$

$$W = -(U_2 - U_1) \Rightarrow$$

$$W = -\Delta U$$

Βολή προς τα πάνω



h; $\vec{v}_0$ = γνωστή

A: $U = 0$ $\quad K = \frac{1}{2} m v^2$

B: $U = mgh$ $\quad K = 0$

$W = \Delta K$ (A) ή $W = -\Delta U$ (B)

$-mg \cdot h = 0 - \frac{1}{2} m v^2 \Rightarrow h = \frac{v^2}{2g}$

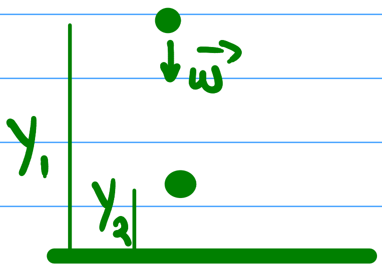
(A), (B) → $\Delta K = -\Delta U \Rightarrow$

$K_2 - K_1 = -(U_2 - U_1)$

$0 - \frac{1}{2} m v^2 = -(mgh - 0)$

$h = \frac{v^2}{2g}$

$K_1 + U_1 = K_2 + U_2$



$y_1 > y_2$

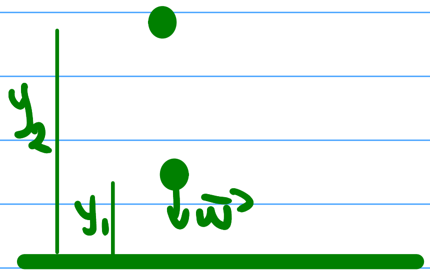
$$W = mg \cdot s \cdot \cos 0^\circ$$

$$W = mg\,|y_1 - y_2| \cdot 1$$

$$W = mg(y_1 - y_2) \Rightarrow$$

$$W = -mg y_2 + mg y_1 \Rightarrow$$

$$W = -\Delta U$$



$y_1 < y_2$ $\quad s = |y_1 - y_2|$

$$W = mg s \cdot \cos 180^\circ$$

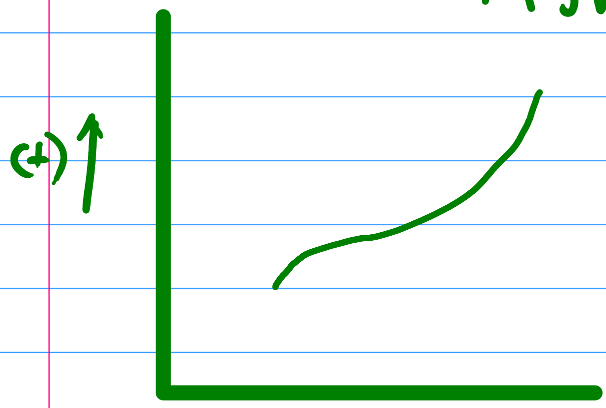
$$W = mg\,|y_1 - y_2|\,(-1)$$

$$W = mg(y_2 - y_1)(-1)$$

$$W = -mg y_2 + mg y_1$$

$$W = -\Delta U$$

Βαρυτική δυναμική ενέργεια κάτα μήκος καμπύλης τροχιάς



$$\Delta\vec{s} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$W_{\beta\acute{\alpha}\rho o \upsilon \varsigma} = \vec{w}\cdot\Delta\vec{s}$$

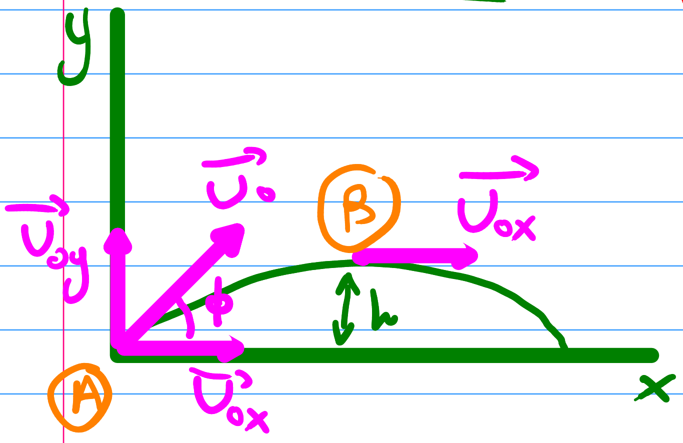
$$\vec{w} = -mg\,\hat{j}$$

$$W_w = -mg\,\hat{j}\,(\Delta x\cdot\hat{i} + \Delta y\,\hat{j})$$

$$W_w = -mg\Delta y \Rightarrow W_w = -\Delta U$$

Πλάγια βολή

$v_0^2 = v_x^2 + v_y^2$



Μέγιστο ύψος

(A) $K = \frac{1}{2} m v_0^2 \quad U = 0$

(B) $K = \frac{1}{2} m v_{x0}^2 \quad U = mgh$

Ενεργεί μόνο το βάρος

$$K_A + U_A = K_B + U_B$$

$$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v_{x0}^2 + mgh \Rightarrow$$

$$v_0^2 - v_{x0}^2 = 2gh \Rightarrow h = \frac{v_0^2 - (v_0 \cdot \cos\phi)^2}{2g}$$

$$v_{0y}^2 = 2gh$$

Συντηρητικές κ' μη συντηρητικές δυνάμεις