



#### Cryptography Lecture 3

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#### **CRYPTOGRAPHIC HASH FUNCTIONS**

#### Hash functions

#### ✓ no secret parameters

- ✓ input string x of arbitrary length  $\Rightarrow$  output h(x) of fixed length n (bits)
- ✓ computation "easy"
- ✓ One-way functions

This is an input to a cryptographic hash function. The input is a very long string, that is reduced by the hash function to a string of fixed length. There are additional security conditions: it should be very hard to find an input hashing to a given value (a preimage) or to find two colliding inputs (a collision).



#### Cryptographic properties



#### preimage



#### Second preimage

#### 2<sup>nd</sup> preimage



- Can be used to protect the integrity of data x
- A secure channel is needed to send h(x) to the verifier.
- The attacker wants to modify x and remain undetected
- The attack is successful if the attacker can find a second preimage of x

## collision



The hacker prepares two versions of a software Let

- 1. x be the correct code
- 2. x' contain a backdoor that gives hacker access to a machine

The hacker submits x for inspection to Bob

If Bob is satisfied, he digitally signs h(x) with his private key The hacker distributes x'; The users verify the signature with Bob's public key This signature works for x and for x', since h(x) = h(x')!

#### Birthday paradox

 the birthday problem or birthday paradox concerns the probability that, in a set of *n* randomly chosen people, some pair of them will have the same birthday.

Example: lets assume that we have a group of 23 people.

$$\binom{23}{2} = \frac{23!}{21!2!} = 253 \, pairs$$

We can show that the birthday paradox is larger than 50%!

#### Birthday paradox

$$p(n) = 1 - \frac{n! \binom{365}{n}}{365^n}$$

n	p(n)	
10	11.7%	
20	41.1%	
23	50.7%	
30	70.6%	
50	97.0%	
57	99.0%	
100	99.99997%	
200	99.999999999999999999 99999999999998%	
300	(100 - (6×10 <sup>-80</sup> ))%	
350	(100 - (3×10 <sup>-129</sup> ))%	
365	(100 – (1.45×10 <sup>-155</sup> ))%	
367	100%	

#### Birthday Attack

- A birthday attack is a name used to refer to a class of brute-force attacks. More precisely,
- "If some function, when supplied with a random input, returns one of |k| equally-likely values, then by repeatedly evaluating the function for different inputs, we expect to obtain the same output after about 1.2|k|<sup>1/2</sup>. "

 Example: for the birthday paradox, we have |k|=365.

#### Brute force

- multiple target second preimage (1 out of many):
- if one can attack 2<sup>t</sup> simultaneous targets, the effort to find a single preimage is 2<sup>n-t</sup>
- multiple target second preimage (many out of many):
  - time-memory trade-off with Θ(2<sup>n</sup>) precomputation and storage Θ(2<sup>2n/3</sup>) time per (2nd) preimage: Θ(2<sup>2n/3</sup>) [Hellman'80]
- answer: randomize hash function with a parameter S

(salt, key, spice,...)

#### Brute force attacks in practice

- (2nd) preimage search
  - n = 128: 23 B\$ for 1 year if one can attack 240 targets
     in parallel
- parallel collision search: small memory using cycle finding algorithms (distinguished points)
   n = 128: 1 M\$ for 8 hours (or 1 year on 100K PCs)
  - n = 160: 90 M\$ for 1 year
  - need 256-bit result for long term security (30 years or more)

#### Quantum era

- in principle exponential parallelism
  - inverting a one-way function: 2<sup>n</sup> reduced to 2<sup>n/2</sup>
     [Grover'96]
- collision search:
- 2<sup>n/3</sup> computation + hardware [Brassard-Hoyer-Tapp'98]
- [Bernstein'09] classical collision search requires  $2^{n/4}$  computation and hardware (= standard cost of  $2^{n/2}$ )

#### Properties in practice

- collision resistance is not always necessary
- other properties are needed:
  - PRF: pseudo-randomness if keyed (with secret key)
  - PRO: pseudo-random oracle property (formalization of security properties when there is no key)
  - near-collision resistance
  - partial preimage resistance (most of input known)
    multiplication freeness
- how to formalize these requirements and the relation between them?

#### **BASIC CONSTRUCTIONS**

#### A simple approach

#### Divide the message into t blocks x<sub>i</sub> of n bits each



#### Merkle–Damgård construction

- f is a compression function
- How to choose the function
- - ad hoc
- based on a block cipher



#### Iterated structure -attack

- iterating f can degrade its security
  - trivial example: 2<sup>nd</sup> preimage



#### Merkle-Damgard strengthening

Algorithm MD-strengthening

Before hashing a message  $x = x_1 x_2 \dots x_t$  (where  $x_i$  is a block of bitlength r appropriate for the relevant compression function) of bitlength b, append a final length-block,  $x_{t+1}$ , containing the (say) right-justified binary representation of b. (This presumes  $b < 2^r$ .)

#### Security relation between f and h

- solution: Merkle-Damgård (MD) strengthening
  - fix IV, use unambiguous padding and insert length at the end
- f is collision resistant ⇒ h is collision resistant [Merkle'89-Damgård'89]
- f is ideally 2<sup>nd</sup> preimage resistant preimage resistant [Lai-Massey'92]
- property preservation has been a heavily studied topic since 2005

#### How (NOT) to strengthen a hash function?[Joux'04]

- answer: concatenation
- h<sub>1</sub> (n1-bit result) and h<sub>2</sub> (n2-bit result)

- intuition: the strength of g against collision/(2<sup>nd</sup>) preimage attacks is the product of the strength of h<sub>1</sub> and h<sub>2</sub>
  - if both are "independent"
- but.... for iterated hash functions only the strongest function matters



# Multi-collisions [Joux '04]

- finding multi-collisions for an iterated hash function is not much harder than finding a single collision (if the size of the internal memory is n bits)
  - algorithm
    - generate R = 2<sup>n1/2</sup>-fold multi-collision for h<sub>2</sub>
    - in R: search by brute force for h<sub>1</sub>





## Multi-collisions [Joux '04]

consider  $h_1$  (n1-bit result) and  $h_2$  (n2-bit result), with n1  $\ge$  n2.

concatenation of 2 iterated hash functions (g(x)= h<sub>1</sub>(x) || h<sub>2</sub>(x)) is as most as strong as the strongest of the two (even if both are independent)

cost of collision attack against g at most

 $n1 \cdot 2^{n2/2} + 2^{n1/2} << 2^{(n1 + n2)/2}$ 

- cost of (2nd) preimage attack against g at most
   n1. 2<sup>n2/2</sup> + 2<sup>n1</sup> + 2<sup>n2</sup> << 2<sup>n1 + n2</sup>
- if either of the functions is weak, the attacks may work better

#### Improving MD iteration

salt + output transformation + counter + wide pipe



## Improving MD iteration

- degradation with use: salting (family of functions, randomization)
- or should a salt be part of the input?
- PRO: strong output transformation g
- also solves length extension
- long message 2nd preimage: preclude fix points
- counter f  $\rightarrow$  fi [Biham-Dunkelman'07]
- multi-collisions, herding: avoid breakdown at 2<sup>n/2</sup> with larger internal memory: known as wide pipe
- e.g., extended MD4, RIPEMD, [Lucks'05]

#### Merkle Tree

 Hash trees allow efficient and secure verification of the contents of large data structures





#### **COMPRESSION FUNCTIONS**

#### Block cipher based



#### Security Analysis

- The security of the Davies–Meyer construction in the Ideal Cipher Model
- For Matyas–Meyer–Oseas construction there is second preimage attack
- For Miyaguchi–Preneel construction there is second preimage attack

#### Non block cipher based

• Sponge construction!







## Motivation for use of a larger permutation key plaintext KS round DP round KS round DP round



#### Motivation for use of a larger permutation k cipher used as X H<sub>1</sub>



# Motivation for use of a larger permutation



#### Motivation for use of a larger







If H1 has r bits (rate) and H2 has c bits (capacity) and the permutation π is "ideal", then a sponge function has security O(2<sup>c</sup>) against (2<sup>nd</sup>) preimage attacks and O(2<sup>c/2</sup>) against collision attacks

# Iteration modes and compression functions

security of simple modes well understood
 – powerful tools available

- analysis of slightly more complex schemes very difficult
  - which properties are meaningful?
  - which properties are preserved?
  - MD versus sponge is still open debate



#### **CONSTRUCTIONS**

#### MD4 family



#### timeline

- 1990: MD4 by Ron Rivest
- 1991: MD5 by Ron Rivest (RFC 1321, 1992)
- 1992: RIPEMD by H. Dobbertin, A. Bosselaers and B. Preneel
- 1993: SHA-0 by U.S. Government (FIPS PUB 180)
- 1995: SHA-1 by U.S. Government (FIPS PUB 180-1)
- 2000: Whirlpool by V. Rijmen and P. Barreto
- 2001: SHA-2 by U.S. Government (FIPS PUB 180-2)
- 2005: First attacks against SHA-1
- 2015: SHA-3 by the Keccak team (FIPS 202)
- 2017: February 2017, CWI Amsterdam and Google announced they had performed a collision attack against SHA-1

#### SHA-3 competition

SHA-3: 224, 256, 384, and 512-bit message digests



#### The candidates



From B. Preneel slides Slides credits: Christophe De Cannière

#### **Round-2 candidates**



#### SHA-3 finalists

- ✓ BLAKE (Aumasson et al.)
- ✓ Grøstl (Knudsen et al.)
- ✓ JH (Hongjun Wu)
- ✓ Keccak (Keccak team, Daemen et al.)
- ✓ Skein (Schneier et al.)
  - Geography: 3 from Europe, 1 from Asia, 1 from America
  - Team members also AES finalist: 3

#### Hardware: post-place & route results ASIC 130nm



#### Keccak: FIPS 202 (published: 5 August 2015)

- append 2 extra bits for domain separation to allow
  - flexible output length (XOFs or eXtendable Output Functions)
  - tree structure (Sakura) allowed by additional encoding
- 6 versions
  - SHA3-224: n=224; c = 448; r = 1152 (72%)
  - SHA3-256: n=256; c = 512; r = 1088 (68%)
  - SHA3-384: n=384; c = 768; r = 832 (52%)
  - SHA3-512: n=512; c = 1024; r = 576 (36%)
  - SHAKE128: n=x; c = 256; r = 1344 (84%)
  - SHAKE256: n=x; c = 512; r = 1088 (68%)
- if result has n bits, H1 has r bits (rate), H2 has c bits (capacity) and the permutation  $\pi$  is "ideal":
  - collisions: min  $(2^{c/2}, 2^{n/2})$
  - $2^{nd}$  preimage: min ( $2^{c/2}$ ,  $2^{n}$ )
  - Preimage: min  $(2^c, 2^n)$

#### SHA3 Winner: Keccak



✓ Based on a new design: sponge



- ✓ Design team: Guido Bertoni, Joan Daemen, Michaël Peeters, Gilles Van Assche
- ✓ FIPS PUB 202: SHA-3 Standard: Permutation-Based Hash and
- Extendable-Output Functions
- https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.202.pdf

#### State of the art

	Output	Classification	
Primitive	Length	Legacy	Future
SHA-2	256, 384, 512	$\checkmark$	$\checkmark$
SHA3	$256,\!384,\!512$	$\checkmark$	$\checkmark$
Whirlpool	512	$\checkmark$	$\checkmark$
SHA3	224	$\checkmark$	X
SHA-2	224	$\checkmark$	X
RIPEMD-160	160	$\checkmark$	X
SHA-1	160	X	X
MD-5	128	X	X
RIPEMD-128	128	X	X

#### Other hash functions

- BLAKE2
- Since 2012
- high efficiency that it offers on modern CPUs
- Whirlpool
- Since 2000
- designed by Vincent Rijmen and Paulo S. L. M.
   Barreto
- 512 bits

