Εισαγωγή στους Φασιθέτες

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Sinusoidal Voltage

$$v(t) = V_m \sin(\omega t + \phi)$$

where V_m is the amplitude of the sinusoid

ω is the angular frequency in rad/s

ϕ is the phase angle in degrees

$\omega t + \phi$ is the argument (όρισμα) of the sinusoid

Period and Frequency

T is the period of a sinusoid; units are seconds

f is the frequency with units of Hz (cycles per second)

$$T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T}$$

Phase between Cosine and Sine

$$v_1(t) = 6V \sin(20t + 40^o)$$
$$v_2(t) = -4V \cos(20t + 20^o)$$

$$v_1(t) = 6V \cos(20t + 40^o - 90^o) = 6V \cos(20t - 50^o)$$
$$v_2(t) = 4V \cos(20t + 20^o - 180^o) = 4V \cos(20t - 160^o)$$

Phase angle between them is 110^o and v_1 leads v_2

Alternatively

$$v_1(t) = 6V \sin(20t + 40^o)$$

$$v_2(t) = -4V \cos(20t + 20^o)$$

$$v_1(t) = 6V \sin(20t + 40^o)$$

$$v_2(t) = 4V \sin(20t + 20^o - 90^o) = 4V \sin(20t - 70^o)$$

Phase angle between them is 110^o

Conversions for Sinusoids	
$A\sin(\omega t + \phi)$	$A\cos(\omega t + \phi - 90^o)$
$-A\sin(\omega t + \phi)$	$A\sin(\omega t + \phi + 180^o)$ Or $A\sin(\omega t + \phi - 180^o)$
$-A\cos(\omega t + \phi)$	$A\cos(\omega t + \phi + 180^o)$ Or $A\cos(\omega t + \phi - 180^o)$
$A\sin(\omega t + \phi)$	$A\sin(\omega t + \phi - 360^o)$ Or $A\sin(\omega t + \phi + 360^o)$
$A\cos(\omega t + \phi)$	$A\cos(\omega t + \phi - 360^o)$ Or $A\cos(\omega t + \phi + 360^o)$

Steps to Perform Before Comparing Angles between Signals

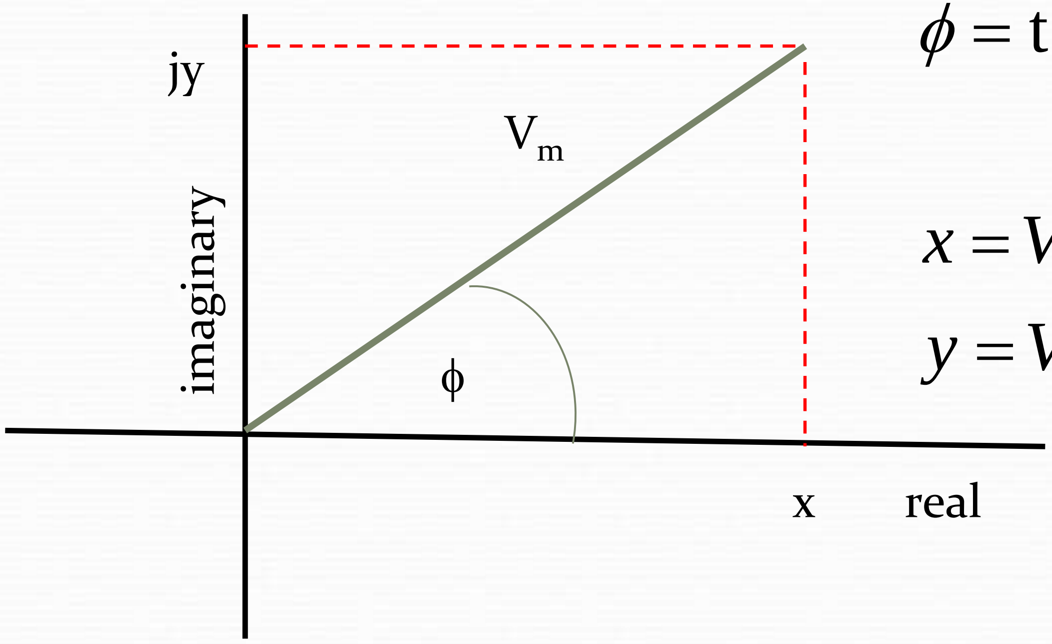
- The comparison can only be done if the angular frequency of both signals are equal.
- Express the sinusoidal signals as the same trig function (either all sines or cosines).
- If the magnitude is negative, modify the angle in the trig function so that the magnitude becomes positive.
- If there is more than 180^o difference between the two signals that you are comparing, rewrite one of the trig functions
- Subtract the two angles to determine the phase angle.

Phasor

- A complex number that represents the amplitude and phase of a sinusoid

$$V_m = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x) = \arctan(y/x)$$



$$x = V_m \cos(\phi)$$

$$y = V_m \sin(\phi)$$

Real Number Line

- If there is no imaginary component to the phasor, then the phasor lies on the real number line (x-axis).
  - Positive real numbers are written as:
    - Phasor notation $P_m \angle 0^o$
    - Rectangular coordinates P_m
  - Negative real numbers are written as:
    - Phasor notation $P_m \angle -180^o$
    - Rectangular coordinates $-P_m$

Imaginary Number Line

- If there is no real component to the phasor, then the phasor lies on the imaginary number line (y-axis).
  - Positive imaginary numbers are written as:
    - Phasor notation $P_m \angle 90^o$
    - Rectangular coordinates jP_m
  - Negative imaginary numbers are written as:
    - Phasor notation $P_m \angle -90^o$
    - Rectangular coordinates $-jP_m$

Phasor Representation

- Polar coordinates: $V = V_m \angle \phi$
- Rectangular coordinates $V = V_m \left[\cos(\phi) + j\sin(\phi) \right]$
  - Sum of sines and cosines $x = V_m \cos(\phi) \quad y = V_m \sin(\phi)$
- Exponential form: $V = V_m e^{j\phi}$

Where the sinusoidal function is:

$$v(t) = V_m \cos(\omega t + \phi)$$

Sinusoid to Phasor Conversion

- The sinusoid should be written as a cosine.
- Amplitude or magnitude of the cosine should be positive.
  - This becomes the magnitude of the phasor
- Angle should be between $+180^o$ and -180^o.
  - This becomes the phase angle of the phasor.
- Note that the frequency of the sinusoid is not included in the phasor notation. It must be provided elsewhere.
  - Phasors are commonly used in power systems, where the frequency is understood to be 60 Hz in the United States.

Sinusoid-Phasor Transformations

Time Domain	Phasor Domain
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle (\phi - 90^o)$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle (\theta - 90^o)$

Assumes V_m is positive and $-180^o \leq \phi \leq 180^o$

Phasor Notation

Phasor notation is used when there are one or more ac power sources in a circuit. All of these power sources operate at the same single frequency.

Used extensive in power systems because almost all of these systems operate at 60 Hz in the United States.

Bold **V** and **I** are used to show that phasor notation is being used.

Examples

Sinusoidal Function:

$$3V\sin(100t+20^0)=3V\cos(100t-70^0)$$

Converting to phasor notation: $3V\angle-70^0$

Sinusoidal Function:

$$7A\sin(350t-100^0)=7A\cos(350t-190^0)$$

$$=-7A\cos(350t-10^0)=7A\cos(350t+170^0)$$

Converting to phasor notation: $7A\angle170^0$

Examples

Rectangular Coordinates	Phasor Notation
$(5+3j)V$	$5.83V\ \angle 31.0^0$
$(-30+j100)A$	$104\text{A}\ \angle -73.3^0$
$(-0.4-0.25j)\Omega$	$0.472\Omega\ \angle 32.0^0$
$(75-j150)\text{A}$	$168\text{A}\ \angle -63.4^0$

Summary

- Phasor notation is used in circuits that have only ac power sources that operate at one frequency.
  - The frequency of operation is not included in the notation, but must be stated somewhere in the circuit description or schematic.
  - The steps to convert between sinusoidal functions and rectangular coordinates were described.
  - To express a phasor $P_m \angle \phi$ in rectangular coordinates (Re + jIm) can be performed using the following equations:

$$P_m = \sqrt{\mathrm{Re}^2 + \mathrm{Im}^2} \qquad \mathrm{Re} = P_m \cos(\phi)$$

$$\phi = \tan^{-1}(\mathrm{Im}/\mathrm{Re}) \qquad \mathrm{Im} = P_m \sin(\phi)$$