THEORY

1 o

y' = y - t, t \in[0.5, 0.6], y (t) = -1 + 2 e^t - t

```
Clear[t]; Clear[y]
Print["Theoretical solution y(t)=",
 DSolve[{y'[t] == y[t] - 2 t, y[0] == 1}, y[t], t] ]
```

Theoretical solution y(t)={{y[t] → 2 - e^t + 2 t}}

RK3

```
f[t_, y_] := y - 2 t;
g[t_] := 2 - Exp[t] + 2 t;
a = 0.5; b = 0.6; n = 1; l = (b - a)/n; t = a; y = g[a];
x2 = N[g[t]]; x3 = Abs[y - x2];
Print["Initial value y0 = ", x2];
Do[Print["STEP  : ", i];
      k1 = f[t, y]; Print["k1 = ", N[k1, 5]];
      k2 = f[t + l/2, y + (l k1)/2]; Print["k2 = ", N[k2, 5]];
      k3 = f[t + l, y - l * k1 + 2 l k2]; Print["k3 = ", N[k3
      x = y + 1/6 l (k1 + 4 k2 + k3);
      t = t + l;
      t1 = N[t];
      x1 = N[x];
      y = x; x2 = N[g[t]]; x3 = Abs[x1 - x2];
      Print["time : ", t1, " num : ", x1, " th : ", x2
```

```
Initial value y0 = 1.351279
STEP  : 1
k1 = 0.3512787
k2 = 0.2688427
k3 = 0.1699194
time : 0.6 num : 1.377888
  th : 1.377881 error : 7.009388×10^-6
```

2 o

i)

Theoretical value

```
Clear[x]; Clear[y]; Clear[z];
Solve[{2 x - y == 3, 3 x + 4 y == 10}, {x, y}]
```

```
{{x → 2, y → 1}}
```

```
n = 3; x = 0.5; y = 0.5;
f1[y_, z_] := (3 + y) / 5;
f2[x_, z_] := (5 - x) / 2;
Print["Gauss-Seidel"]
Do[x1 = f1[y, z]; y1 = f2[x1, z];
 Print[i, "   ,   ", N[x1, 3], "   ,   ", N[y1, 3]];
 x = x1; y = y1, {i, 1, n}]
```

```
Gauss-Seidel
1   ,   0.7   ,   2.15
2   ,   1.03   ,   1.985
3   ,   0.997   ,   2.0015
```

ii)

3 / 8 COMPOSITE SIMPSON

```
Clear[x]; f[x_] := Log[1 + x^2];
thv = Integrate[f[x], {x, 0, 0.9}]
Print["Theoretical value = "]
a = 0; b = 0.9; n = (b - a)/0.1; h = 0.1;
S0 = N[f[a]] + N[f[b]]; S1 = 0; S2 = 0; S3 = 0;
Print[0, "  ,  ", "  x=  ", a, "  ’  ", f[a]]
Do[y = a + i * h; S1 += N[f[y]]; Print[i, "  ,  ", "  x=  ",
Do[y = a + i * h; S2 += N[f[y]]; Print[i, "  ,  ", "  x=  ",
Do[y = a + i * h; S3 += N[f[y]]; Print[i, "  ,  ", "  x=  ",
Print[n, "  ,  ", "  x=  ", b, "  ’  ", f[b]]
S = 3/8 h (S0 + 3 S1 + 3 S2 + 2 S3);
Print["Composite 3/8 Simpson I(f)=", N[S], "    Absolu
0.1996244
Theoretical value =
0  ,    x=  0  ’  0
1  ,    x=  0.1  ’  0.009950331
4  ,    x=  0.4  ’  0.14842
7  ,    x=  0.7  ’  0.3987761
2  ,    x=  0.2  ’  0.03922071
5  ,    x=  0.5  ’  0.2231436
8  ,    x=  0.8  ’  0.4946962
3  ,    x=  0.3  ’  0.0861777
6  ,    x=  0.6  ’  0.3074847
9.  ,    x=  0.9  ’  0.5933268
Composite 3/8 Simpson I(f)=0.1996227
     Absolute error   =1.644141 × 10^-6
```

3 o

i)

```
Clear[x]
z = InterpolatingPolynomial[
   {{1.5, 2.5}, {2, 3}, {2.2, 3.5}}, x]
Expand[
 z]
```

$$3.5 + (1.428571 + 2.142857\ (-1.5 + x))\ (-2.2 + x)$$

$$7.428571 - 6.5\ x + 2.142857\ x^2$$

ii)

Theoretical value

```
Print["Equation : ", Expand[(x^2 + 1) (x - 2)], " = 0"]
Print["Theoretical solution ; ",
 NSolve[-2 + x - 2 x^2 + x^3 == 0, x]]
```

Equation : $-2 + x - 2\ x^2 + x^3 = 0$

Theoretical solution ;

$\{\{x \to 0. - 1.\ i\}, \{x \to 0. + 1.\ i\}, \{x \to 2.\}\}$

```
Print["Newton Method"]
f[x_] := x^3 - 2 x^2 + x - 2
g[x_] := 3 x^2 - 4 x + 1
x = 1.8;
Do[y = x - f[x] / g[x]; z = N[y];
 Print[i, "   ,   ", z]; x = y, {i, 1, 4}]
```

Newton Method

1 , 2.040909

2 , 2.001281

3 , 2.000001

4 , 2.

LABORATORY

1 o
Least Squares degree n = 1

```
Clear[x];
data = {{1, 2.5}, {1.5, 2.5}, {2, -1.5}, {2.8, 1}};
Fit[data, {1, x}, x]
```

3.35785 − 1.223479 x

2 o

Theoretical Value

```
Clear[x]; f[x_] := Log[1 + x^2];
thv = Integrate[f[x], {x, 0, 0.9}];
Print["Theoretical value = ", thv]
Print[" "];
```

```
Theoretical value = 0.1996244
```

Numerical Value

COMPOSITE TRAPEZOIDAL

```
Clear[f];
f[x_] := Log[1 + x^2];
h = 0.1; a = 0; b = 0.9; n = (b - a)/h
TV = NIntegrate[f[x], {x, a, b}];
Print["f(", a, ")    = ", f[a]];
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0; Sb1 = 0; Sb2 = 0;
Do[a += h; Str1 += f[a]; Print["f(", a, ") = ", f[a]], {i,
Print["f(", a + h, ") = ", f[b]];
Str = 1/2 h (Str0 + 2 Str1);
Print["Composite Trapezoidal I(f)=", N[Str], "    Absc
9.
```

```
f(0)    = 0
f(0.1) = 0.009950331
f(0.2) = 0.03922071
f(0.3) = 0.0861777
f(0.4) = 0.14842
f(0.5) = 0.2231436
f(0.6) = 0.3074847
f(0.7) = 0.3987761
f(0.8) = 0.4946962
f(0.9) = 0.5933268
Composite Trapezoidal I(f)=0.2004533
     Absolute error   =0.0008289138
```

```
3 o
y' = y - t = f[t, y],
t \in[0.5, 0.6],  y (t) = -1 + 2 e^t - t
f'[t, y] = y' - 1 = (y - t) - 1
```

```
f[t_, y_] := y - 2 t;
g[t_] := 2 - Exp[t] + 2 t;
a = 0.5; b = 0.6; l = 0.1; n = 1;
f1[t_, y_] := (y - t) - 1;
t = 0.5; y = g[a];
x2 = N[g[t]]; x3 = Abs[y - x2];
Print["time", "    ,    ", "numerical",
  "    ,    ", "theoretical", "    ,    ", "error"];
Do[x = y + l * f[t, y] + l * l * f1[t, y] / 2;
  t = t + l; t1 = N[t]; x1 = N[x]; y = x;
  x2 = N[g[t]]; x3 = Abs[x1 - x2];
  Print[t1, "     ,    ", x1, "     ,    ",
   x2, "       ,    ", x3], {i, 1, 1}];
```

```
time    ,    numerical    ,    theoretical    ,    error
0.6     ,    1.385663     ,    1.377881       ,    0.007781796
```