8 February 2016

THEORY

1 o

$y' = 1 + y/t, \quad 1 \le t \le 1.1, \quad y(t) = 2t + t \ln t$

Theoretical solution

```
Clear[t]
g[t_] := 2 t + t Log[t];
Simplify[D[g[t], t] - 1 - g[t] / t]
y = g[1];
Print["Initial value y₀ = ", y]
```

```
0
```

```
Initial value y₀ = 2
```

RK4

```
f[t_, y_] := 1 + y / t;
a = 1; b = 1.1; n = 1; l = 0.1; t = 1;
x2 = N[g[t]]; x3 = Abs[y - x2]; y = g[1];
Print["Initial value y0 = ", x2];
Do[Print["STEP  : ", i]; k1 = f[t, y]; Print["k1 = ", N[
    k2 = f[t + l/2, y + l k1/2]; Print["k2 = ", N[k2, 7]];
    k3 = f[t + l/2, y + l k2/2]; Print["k3 = ", N[k3, 7]];
    k4 = f[t + l, y + l k3]; Print["k4 = ", N[k4, 7]];
    x = y + 1/6 l (k1 + 2 k2 + 2 k3 + k4);
    t = t + l;
    t1 = N[t];
    x1 = N[x];
    y = x; x2 = N[g[t]]; x3 = Abs[x1 - x2];
    Print["time : ", t1, ", numerical : ", x1, ", th
```

Initial value y_0 = 2.

STEP : 1

k_1 = 3.000000

k_2 = 3.047619

k_3 = 3.049887

k_4 = 3.095444

time : 1.1, numerical : 2.304841

, theoretical : 2.304841, error : 2.715153×10^{-7}

Taylor order n = 2 :

$$y_{i+1} = y_i + l\, f(t, y) + l\verb|^|2\, f'(t, y) / 2,$$

where :

In[1]:=
```
Clear[t]; Clear[y];
f[t_, y_] := 1 + y / t;
f'[t_, y_] := 0 + (t f[t, y] - y) / t^2
Print["f'(t,y) = ",
 f'[t, y], " = ", Simplify[f'[t, y]]]
```

$$f'(t,y) = \frac{-y + t\left(1 + \frac{y}{t}\right)}{t^2} = \frac{1}{t}$$

2 o

```
Clear[x]
f[x_] := Sqrt[1 + x^4]
th = N[Integrate[f[x], {x, 0, 0.6}]];
Print["Theoretical value : ", th]
```

Theoretical value : 0.6076419

```
P[x_] := InterpolatingPolynomial[
  {{0, f[0]}, {0.3, f[0.3]}, {0.6, f[0.6]}}, x]
num1 = N[Integrate[P[x], {x, 0, 0.6}]];
Print["Interpolating Polynomial : ", Expand[P[x]]]
Print["Approximation : ",
 num1, ",  error = ", Abs[num1 - th]]
```

Interpolating Polynomial : $1. - 0.07776516\,x + 0.3041264\,x^2$

Approximation : 0.6078994, error = 0.0002574261

COMPOSITE SIMPSON

```
Clear[x]
f[x_] := Sqrt[1 + x^4]
n = 6; a = 0; b = 0.6; h = 0.1;
Print[a, "  ,  ", f[a]]
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0; Sb1 = 0; Sb2 = 0;
Do[a += h; If[EvenQ[i], Sb2 += f[a], Sb1 += f[a]]; Print[a
Print[b, "  ,  ", f[b]]
Sb = 1/3 h (Sb0 + 4 Sb1 + 2 Sb2);
Print["Composite Simpson I(f)=", N[Sb], "     Absolute
```

```
0  ,  1
0.1  ,  1.00005
0.2  ,  1.0008
0.3  ,  1.004042
0.4  ,  1.012719
0.5  ,  1.030776
0.6  ,  1.062826
Composite Simpson I(f)=0.6076446
     Absolute error    =2.616439×10^-6
```

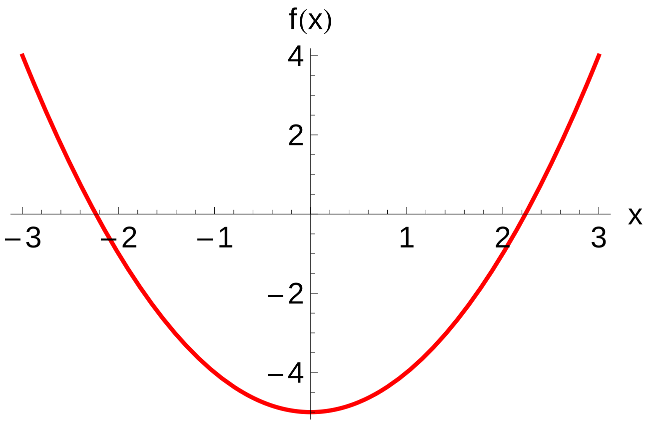
3 o

ii)

```
Clear[x]
Print["Theoretical solution : ",
 NSolve[x^2 - 5 == 0, x]]
f[x_] := x^2 - 5
Print["f(x)=", f[x], ",   f'(x)=", D[f[x], x]]
Plot[f[x], {x, -3, 3}, PlotStyle → {Red, Thick},
 AxesLabel → {"x", "f(x)"},
 BaseStyle → {FontFamily → "Arial", FontSize → 14}]
Clear[x]
g[x_] := Simplify[x - f[x] / (2 x)]
Print["Right hand-side: x-f(x)/f'(x)  = ", g[x]]
x = 2;
Print["Initial value : ", x]
Do[y = g[x];
 Print[i, "    ,    ", N[y, 10]]; x = y, {i, 1, 3}]
```

Theoretical solution : {{x → −2.236068}, {x → 2.236068}}

f(x)=$-5+x^2$,   f′(x)=2 x



Right hand-side: x−f(x)/f′(x) = $\frac{5+x^2}{2x}$

Initial value : 2

1   ,   2.250000000

2   ,   2.236111111

3   ,   2.236067978

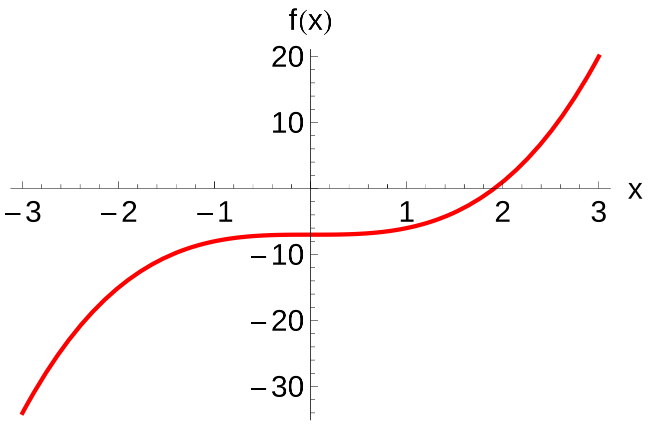
LABORATORY

1 o

```
Clear[x]
Print["Theoretical solution : ",
 NSolve[x^3 - 7 == 0, x]]
f[x_] := x^3 - 7
Print["f(x)=", f[x], ",   f′(x)=", D[f[x], x]]
Plot[f[x], {x, -3, 3}, PlotStyle → {Red, Thick},
 AxesLabel → {"x", "f(x)"},
 BaseStyle → {FontFamily → "Arial", FontSize → 14}]
Clear[x]
g[x_] := Simplify[x - f[x] / (3 x^2)]
Print["Right hand-side: x-f(x)/f′(x)  = ", g[x]]
x = 1.7;
Print["Initial value : ", x]
Do[y = g[x];
 Print[i, "    ,    ", N[y, 10]]; x = y, {i, 1, 3}]
```

Theoretical solution : {{x → −0.9564656 − 1.656647 i}, {x → −0.9564656 + 1.656647 i}, {x → 1.912931}}

$f(x) = -7 + x^3$, $f'(x) = 3 x^2$



Right hand-side: x-f(x)/f′(x) = $\frac{7}{3 x^2} + \frac{2 x}{3}$

Initial value : 1.7

1   ,   1.940715

2   ,   1.913327

3   ,   1.912931

2 o

Theoretical Value

```
Clear[x]
f[x_] := Sqrt[1 + x^2]
tv = Integrate[f[x], {x, 0, 0.5}];
Print["Theoretical value = ", th]; Print[" "];
```

Theoretical value = 0.5201144

Numerical Value

COMPOSITE TRAPEZOIDAL

```
Clear[f];
f[x_] := Sqrt[1 + x^2];
a = 0; b = 0.5; n = (b - a)/0.1;
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0; Sb1 = 0; Sb2 = 0;
Do[a += h; Str1 += f[a], {i, 1, n - 1}]
Str = 1/2 h (Str0 + 2 Str1);
Print["Composite Trapezoidal I(f)=", N[Str], "     Absc
```

Composite Trapezoidal I(f)=0.5204872
Absolute error =0.000372798

3 o
$y' = 1 + y/t,\ 1 \le t \le 1.1,\ y(t) = 2t + t \ln t$

Theoretical solution

```
Clear[t]
g[t_] := 2 t + t Log[t];
Simplify[D[g[t], t] - 1 - g[t] / t]
Print["Initial value y₀ = ", N[g[1]]]
```

0

Initial value y_0 = 2.

RK3

```
Clear[f]; Clear[g];

g[t_] := 2 t + t Log[t]; f[t_, y_] := 1 + y / t; a = 1; b = 1.1;

Print["time step : ", l]
x2 = N[g[t]]; x3 = Abs[y - x2]; Print["Initial value y0 =
Do[Print["STEP  : ", i]; k1 = f[t, y]; Print["k1 = ", N[
      k2 = f[t + l/2, y + (l k1)/2]; Print["k2 = ", N[k2, 7]];

      k3 = f[t + l, y - l * k1 + 2 l k2]; Print["k3 = ", N[k3

      t = t + l; t1 = N[t]; x1 = N[x, 7]; y = x; x2 = N[g[t]];
      Print["time : ", t1, " num : ", x1, " th : " , x2
```

time step : 0.1

Initial value y_0 = 2.

STEP : 1

k_1 = 3.000000

k_2 = 3.047619

k_3 = 3.099567

time : 1.1 num : 2.304834

th : 2.304841 er : 7.142951×10^{-6}