17 June 2016

THEORY

1 o

$y' = 1 - y + t,\ 1 \le t \le 1.1,\ y(t) = t + \text{Exp}[-t]$

Theoretical solution

```
Clear[t]
g[t_] := t + Exp[-t];
Simplify[D[g[t], t] - 1 + g[t] - t]
y = g[1];
Print["Initial value y₀ = ", y, " ≃ ", N[y]]
0
```

Initial value $y_0 = 1 + \frac{1}{e} \simeq 1.367879$

RK4

```
f[t_, y_] := 1 - y + t;
a = 1; b = 1.1; n = 1; l = 0.1; t = 1;
x2 = N[g[t]]; x3 = Abs[y - x2]; y = g[1];
Print["Initial value y0 = ", x2];
Do[Print["STEP  : ", i]; k1 = f[t, y]; Print["k1 = ", N[
      k2 = f[t + l/2, y + l k1/2]; Print["k2 = ", N[k2, 7]];
      k3 = f[t + l/2, y + l k2/2]; Print["k3 = ", N[k3, 7]];
      k4 = f[t + l, y + l k3]; Print["k4 = ", N[k4, 7]];
      x = y + 1/6 l (k1 + 2 k2 + 2 k3 + k4);
      t = t + l;
      t1 = N[t];
      x1 = N[x];
      y = x; x2 = N[g[t]]; x3 = Abs[x1 - x2];
      Print["time : ", t1, ", numerical : ", x1, ", th
```

```
Initial value y0 = 1.367879
STEP  : 1
k1 = 0.6321206
k2 = 0.6505145
k3 = 0.6495948
k4 = 0.6671611
time : 1.1, numerical : 1.432871
 , theoretical : 1.432871, error : 3.015289×10^-8
```

2 o

```
Clear[x]
f[x_] := Sqrt[1 + x^2]
th = N[Integrate[f[x], {x, 0, 0.4}]];
Print["Theoretical value : ", th]
```

```
Theoretical value : 0.4104243
```

```
P[x_] := InterpolatingPolynomial[
  {{0, f[0]}, {0.2, f[0.2]}, {0.4, f[0.4]}}, x]
num1 = N[Integrate[P[x], {x, 0, 0.4}]];
Print["Interpolating Polynomial : ", Expand[P[x]]]
Print["Approximation : ",
 num1, ",  error = ", Abs[num1 - th]]
```

```
Interpolating Polynomial : 1. +
 0.005456624 x + 0.4678144 x^2
```

```
Approximation : 0.4104166,  error = 7.680253 × 10^-6
```

COMPOSITE TRAPEZOIDAL

```
Clear[x]
f[x_] := Sqrt[1 + x^2]
n = 4; a = 0; b = 0.4; h = 0.1;
TV = NIntegrate[f[x], {x, a, b}];
Print["Theoretical value =", TV]; Print[" "];
Str0 = f[a] + f[b]; Sb0 = Str0; Str1 = 0;
Do[a += h; Str1 += f[a], {i, 1, n - 1}]
Str = 1/2 h (Str0 + 2 Str1);
Print["Composite Trapezoidal I(f)=", N[Str], "    Absc
```

```
Theoretical value =0.4104243

Composite Trapezoidal I(f)=0.4107339
     Absolute error   =0.0003096075
```

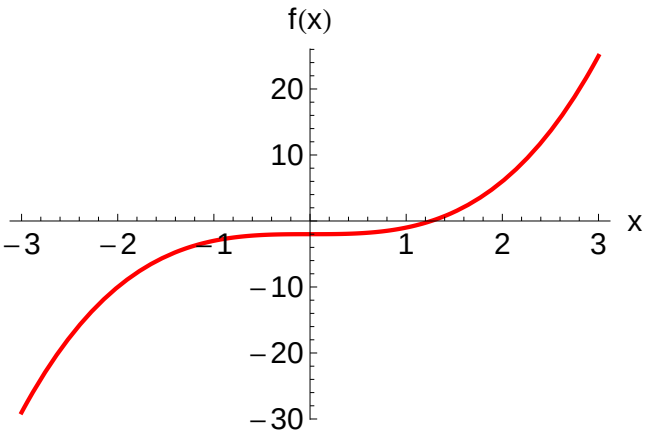
3 o

ii)

```
Clear[x]
Print["Theoretical solution : ",
 NSolve[x^3 - 2 == 0, x]]
f[x_] := x^3 - 2
Print["f(x)=", f[x], ",  f'(x)=", D[f[x], x]]
Plot[f[x], {x, -3, 3}, PlotStyle → {Red, Thick},
 AxesLabel → {"x", "f(x)"},
 BaseStyle → {FontFamily → "Arial", FontSize → 14}]
Clear[x]
g[x_] := Simplify[x - f[x] / (3 x^2)]
Print["Right hand-side: x-f(x)/f'(x) = ", g[x]]
x = 1.2;
Print["Initial value : ", x]
Do[y = g[x];
 Print[i, "   ,   ", N[y, 10]]; x = y, {i, 1, 3}]
```

```
Theoretical solution : {{x → -0.6299605 - 1.091124 i},
  {x → -0.6299605 + 1.091124 i}, {x → 1.259921}}
```

$f(x) = -2 + x^3, \quad f'(x) = 3x^2$



Right hand-side: $x - f(x)/f'(x) = \dfrac{2\left(1 + x^3\right)}{3x^2}$

```
Initial value : 1.2
1   ,   1.262963
2   ,   1.259928
3   ,   1.259921
```