11 July 2013

1 o

i)

In[1]:=
```
ClearAll[x, y, z]
<< VectorAnalysis`;
SetCoordinates[Cartesian[x, y, z]];
F[x_, y_, z_] := {2 x y z, x^2 z, x^2 y}
Print["Curl F = ", Curl[F[x, y, z]]]
```

Curl F = {0, 0, 0}

ii)

In[12]:=
```
f[x_, y_] := Exp[-x] Cos[2 y]
```

In[13]:=
```
Grad[f[x, y]]
f1 = Grad[f[x, y]] /. {x → 0, y → Pi / 8};
Print["grad(f} = ", f1]
a1 = {1 / Sqrt[2], 1 / Sqrt[2], 0};
Print["Unit vector : ", a1]
Print["Directional Derivative : ", f1.a1]
```

Out[13]= $\{-e^{-x} \text{Cos}[2 y], -2 e^{-x} \text{Sin}[2 y], 0\}$

grad(f} = $\left\{-\frac{1}{\sqrt{2}}, -\sqrt{2}, 0\right\}$

Unit vector : $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$

Directional Derivative : $-\frac{3}{2}$

iii)

```
f[x_, y_] := x^2 - x - y^2 + y + 5
D[f[x, y], x]
D[f[x, y], y]
Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]
```

$-1 + 2 x$

$1 - 2 y$

$\left\{\left\{x \to \frac{1}{2}, y \to \frac{1}{2}\right\}\right\}$

```
A = D[D[f[x, y], x], x] /. {x -> 1/2, y -> 1/2}
B = D[D[f[x, y], x], y] /. {x -> 1/2, y -> 1/2}
C1 = D[D[f[x, y], y], y] /. {x -> 1/2, y -> 1/2}
z = A * C1 - B^2
```

2

0

-2

-4

D > 0 and A > 0   minimum

2 o

i)

In[19]:=
```
Integrate[x + 3 y^2, y]
Integrate[x + 3 y^2, {y, 0, x^2}]
Print["Double Integral: ",
 Integrate[x + 3 y^2, {x, 0, 1}, {y, 0, x^2}]]
```

Out[19]= $x\,y + y^3$

Out[20]= $x^3 + x^6$

Double Integral: $\frac{11}{28}$

ii)

In[22]:=
```
DSolve[y''[x] + 2 y'[x] + 5 y[x] == 0, y[x], x]
Print["Partial Solution : ",
 DSolve[{y''[x] + 2 y'[x] + 5 y[x] == 0,
   y'[0] == -1, y[0] == 0}, y[x], x]]
```

Out[22]= {{y[x] → e^{-x} C[2] Cos[2 x] + e^{-x} C[1] Sin[2 x]}}

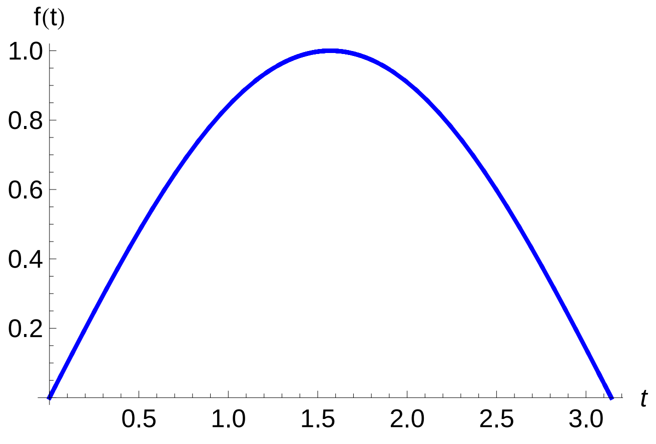
Partial Solution : {{y[x] → $-\frac{1}{2}$ e^{-x} Sin[2 x]}}

3 o

i)

In[40]:=
```
fgr = Plot[Sin[t], {t, 0, Pi}, PlotStyle → Thick,
  ColorFunction -> Function[Blue],
  AxesLabel → {t, "f(t)"},
  BaseStyle → {FontFamily → "Arial", FontSize → 12}]
```

Out[40]=



In[41]:=
```
T = Pi;
a0 = (4 / T) * Integrate[Sin[t], {t, 0, Pi / 2}];
Print["a0 = ", a0]
```

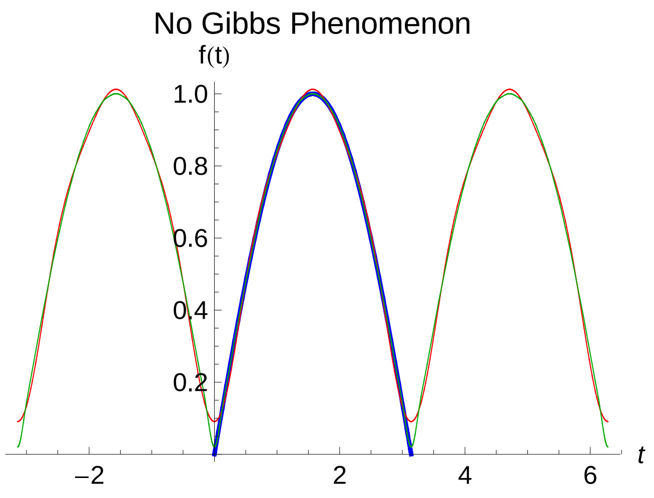
$$a_0 = \frac{4}{\pi}$$

In[44]:=
```
an = (4 / T) * Integrate[
      Sin[t] * Cos[(2 * n * Pi * t) / T], {t, 0, Pi / 2}] /.
    {Cos[n π] → (-1) ^n, Sin[n π] → 0};
Print["an = ", an]
```

$$a_n = \frac{4}{\left(1 - 4 n^2\right) \pi}$$

In[124]:=
```
Table[an, {n, 1, 15}];
S3 = a0 / 2 + Sum[an Cos[2 n t], {n, 1, 3}];
S15 = a0 / 2 + Sum[an Cos[2 n t], {n, 1, 15}];
gf3 = Plot[S3, {t, -Pi, 2 Pi}, PlotRange → All,
    ColorFunction -> Function[Red]];
gf15 = Plot[S15, {t, -Pi, 2 Pi}, PlotRange → All,
    ColorFunction -> Function[Darker[Green]]];
fg = Show[fgr, gf3, gf15, PlotRange → All,
   BaseStyle → {FontFamily → "Arial", FontSize → 12},
   PlotLabel → "No Gibbs Phenomenon"]
```

Out[129]=



ii)

In[32]:=
```
Print["y=ax+b"]
Fit[{{1.2, -0.5}, {1.8, 1.0},
  {2.0, 1.5} {2.5, 2}}, {1, x}, x]
```

y=ax+b

Out[33]= $-1.027157 + 0.8226837\, x$